**2007 Mu Alpha Theta National Convention – Theta Circles Solutions**

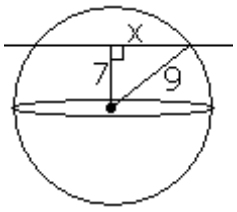
1. D Area = $\pi r^2 = 100\pi$

2. B Largest area will be polygon with the greatest # of sides

3. B Opposite angles of a cyclic quadrilateral are supplementary

4. C Ratio of areas is the square of the ratio of corresponding parts

5. A V=$\frac{4}{3}\pi r^3$ =972, so r = 9. Then, $7^2 + x^2 = 9^2$, and $x^2 = 32$. A = $32\pi$.




6. B .5(94 + 120) = 107, $m\angle 1 = 180 - 107 = 73$

7. C Using similar $\Delta s$, $\frac{16}{12} = \frac{x}{10}$, so 12x = 160, and x = $\frac{40}{3}$

8. B P is a right angle, so in the 30-60-90 triangle, QR = 12, PR = 6,
PQ = $6\sqrt{3}$. Area of triangle = .5(6)($6\sqrt{3}$) = $18\sqrt{3}$.

9. B Center is the perpendicular bisector of two of the chords RS, RT, ST.
Chord RS: Midpt (3, 2), $m = 3$, $m_\perp = \frac{1}{3}$, eq. of $\perp$ bisector: y = $\frac{1}{3}$x + 1
Chord RT: Midpt (1, -2), $m = 1$, $m_\perp = -1$, eq. of $\perp$ bisector: y = -x – 1
Solve system to find center $\left(\frac{-3}{2}, \frac{1}{2}\right)$, 2h + 4k = -3 + 2 = -1

10. A $C = 2\pi r = 4\pi$ for one revolution, # of revs = $\frac{5280}{4\pi} = \frac{1320}{\pi}$

11. D Pythagorean triples 7-24-25 and 15-20-25 give points
$(\pm 7, \pm 24), (\pm 24, \pm 7), (\pm 15, \pm 20), (\pm 20, \pm 15)$, plus the points on the axes
$(0, \pm 25)$ and $(\pm 25, 0)$. Total of 20 points.

12. D C = $12\pi$, so r = 6. 2x + x + 90 + 90 = 360; x = 60, so the segments are

60 and 120 degrees. Segment area = Sector area – area of triangle.

60 degree segment: A = $\frac{60}{360}\left(\pi 6^2\right) - \frac{6^2\sqrt{3}}{4} = 6\pi - 9\sqrt{3}$

120 degree segment: A = $\frac{120}{360}\left(\pi 6^2\right) - \frac{1}{2}(3)(6\sqrt{3}) = 12\pi - 9\sqrt{3}$

Total area = $18\pi - 18\sqrt{3}$, so P = 18, Q = 18, P + Q = 36

13. A $x^2 + y^2 - 6x + 4y - 36 = 0$; $x^2 - 6x + 9 + y^2 + 4y + 4 = 36 + 9 + 4$;
$(x-3)^2 + (y+2)^2 = 49$; center (3, -2) and radius 7
Distance from center to (11, 13) = $\sqrt{(11-3)^2 + (13-(-2))^2} = 17$
Distance to circle = distance to center – radius of circle = 17 – 7 = 10

14. A $\frac{m\overset{\frown}{PUT} - m\overset{\frown}{QS}}{2} = m\angle R$; $\frac{100 - m\overset{\frown}{QS}}{2} = 10$; $m\overset{\frown}{QS} = 80$.
$m\angle U = \frac{1}{2}(80) = 40$.

15. B Each minute, minute hand moves $\frac{360}{60} = 6^\circ$, and hour hand moves
$\frac{30}{60} = 0.5^\circ$. Hands move closer or farther apart at a rate of 6 – 0.5 = 5.5 deg/min.
At 9:00, angle between hands = 30(3) = 90. 24 minutes later, hands are 90 + 24(5.5) = 222 degrees apart. Obtuse angle on other side = 360 – 222 = 138.

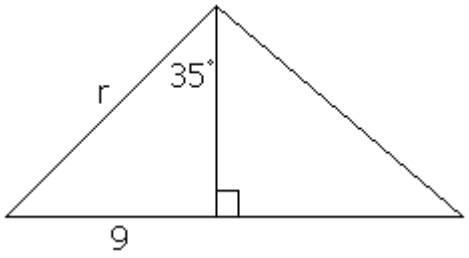
16. B Parabola's vertex inside circle and opens up, passes through circle twice

17. A No. of flippable circular permutations of n items = $\frac{(n-1)!}{2}$

18. B $r_1 + r_2 = 8, r_2 + r_3 = 10, r_1 + r_3 = 12$. So, $r_2 = 8 - r_1$, then substitute:
$8 - r_1 + r_3 = 10$. Add this to third equation to get $8 + 2r_3 = 22$, and $r_3 = 7$.
Substitute into original equations to find other radii 3 and 5.
(3)(5)(7) = 105.

19. B Radius of sector becomes slant height of cone, and arclength becomes circumference. Arclength = $\frac{72}{360}(2\pi \cdot 10) = 4\pi$. So, radius of cone = 2. Then,
$H^2 + 2^2 = 10^2$, $H = \sqrt{96} = 4\sqrt{6}$.

20. A $\sin 35 = \frac{9}{r}$, so $r = \frac{9}{\sin 35}$, and Area = $\pi r^2 = \frac{81\pi}{(\sin 35^\circ)^2}$




21. B revolving creates cylinder with R = 4 and H = 6 with cylindrical hole with r = 1 and H = 6, so V = $\pi(R^2 - r^2)H = \pi(4^2 - 1^2)6 = 90\pi$

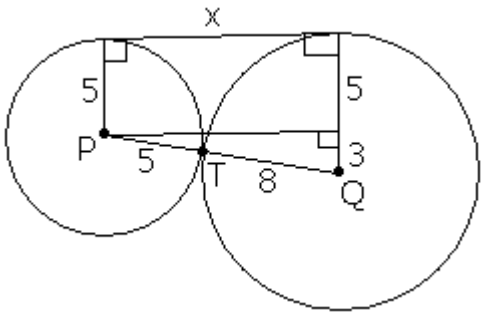
22. B begins as a right angle, then becomes obtuse as P approaches X

23. C each travels the circumference of a circle 3 times per minute
Mike: $3(2\pi \cdot 6) = 36\pi$; Wendy: $3(2\pi \cdot 9) = 54\pi$; difference = $18\pi$

24. A Draw the other diagonal, and the shaded region below is the same area and is a segment of a circle. Segment area = sector – triangle = $\frac{1}{4}\pi \cdot 6^2 - \frac{1}{2} \cdot 6 \cdot 6 = 9\pi - 18$

25. A ecc. = c/a, and as an ellipse approaches a circle, $a \to b$, so $c \to 0$.

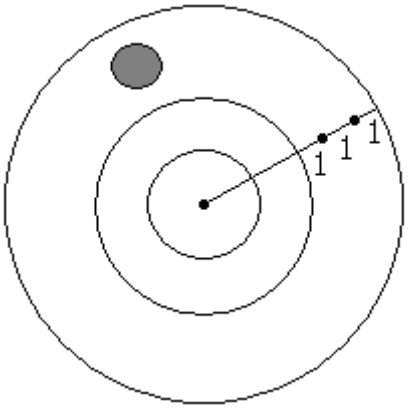
26. B $x^2 + 3^2 = (5+8)^2$, so $x^2 = 160$ and $x = 4\sqrt{10}$




27. E $1 \cdot 6 \cdot 3 \cdot 2 \cdot 2 \cdot 2 \cdot 1 \cdot 1 \cdot 1$ -- the first spot can be anyone, but he can sit anywhere (he is the starting point), then 6 people (from the other families) can sit next to him, then 3 people (from the last family) can sit next, then 2 people from the first family, etc.

28. A The center of the disk must land at least one unit inside the largest circle and at least one unit outside the middle circle, so the center of the disk can land between 4 and 5 units away from the center of the target, which is an area of $\pi(5^2 - 4^2) = 9\pi$. If some part of the disk hits the target, then the center of the disk must be no more than 1 unit away from the outside circle of the target, or no more than 7 units away from the center of the target. This gives an area of $\pi \cdot 7^2 = 49\pi$.

Probability = $\frac{9\pi}{49\pi} = \frac{9}{49}$.




29. D $x(x+5) = 6(6+4)$, $x^2 + 5x - 60 = 0$, $x = \frac{-5 \pm \sqrt{5^2 - 4(1)(-60)}}{2(1)}$,

$x = \frac{-5 + \sqrt{265}}{2} = \frac{1}{2}(-5 + \sqrt{265})$, J = -5, K = 265, $\frac{-K}{J} = \frac{-265}{-5} = 53$

30. A Draw in remainder of original cone, and use similar triangles to find the height, h, of the missing piece. So, $\frac{h}{h+2} = \frac{6}{10}$, which gives h = 3. Then, volume of frustum = volume of large cone – volume of small cone = $\frac{1}{3}\pi \cdot 10^2 \cdot 5 - \frac{1}{3}\pi \cdot 6^2 \cdot 3 = \frac{392\pi}{3}$. Number of grains = $\frac{392\pi}{3} \div \frac{\pi}{6} = 784$.