Circles Theta 2007 Mu Alpha Theta National Convention

Directions: No calculators are allowed on this test. Choice E, "NOTA" stands for "none of the above". Diagrams are not necessarily drawn to scale. All arcs are minor arcs unless otherwise noted.

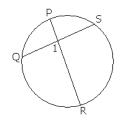
- 1. What is the area of a circle with a radius of 10?
 - A. 10π B. 20π C. 25π D. 100π E. NOTA
- 2. Which of the following regular polygons inscribed in a circle of radius R will have the greatest area?

А.	heptagon	B. nonagon	C. octagon
D.	pentagon	E. NOTA	

- 3. Which statement below is not necessarily true?
 - A. If a parallelogram is inscribed in a circle, then the parallelogram is a rectangle.
 - B. The opposite angles of a cyclic quadrilateral are congruent.
 - C. A line tangent to a circle at point T is perpendicular to the radius of the circle drawn to point T.
 - D. The perpendicular bisector of a chord of a circle passes through the center of the circle.
 - E. NOTA
- 4. The radii of two circles are x and y, with x < y. What is the ratio of the area of the smaller circle to the area of the larger circle?

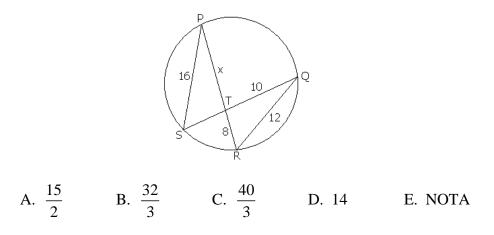
A.
$$\sqrt{\frac{x}{y}}$$
 B. $\frac{x}{y}$ C. $\frac{x^2}{y^2}$ D. $\frac{x^3}{y^3}$ E. NOTA

- 5. A sphere with volume 972π is intersected by a plane 7 units from its center. What is the area of the region which is the intersection of the plane and the sphere?
 - A. 32π B. 130π C. 194π D. 292π E. NOTA
- 6. Find $m \angle 1$ in the circle shown below if $m PQ = 94^{\circ}$ and $mRS = 120^{\circ}$.

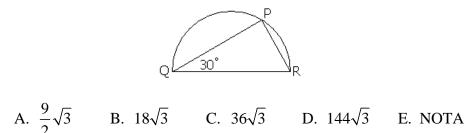


A. 60° B. 73° C. 107° D. 133° E. NOTA

7. Given PS = 16, TR = 8, RQ = 12, and QT = 10, find the length of PT in the circle shown below.



8. In the diagram below, arc QPR defines a semicircle with radius 6, and $m \angle PQR = 30^{\circ}$. What is the area of $\triangle PQR$?



9. A circle centered at point O is circumscribed about ΔRST with vertices R(2, -1), S(4, 5), and T(0, -3). The coordinates of point O are (h, k). What is the value of 2h + 4k?

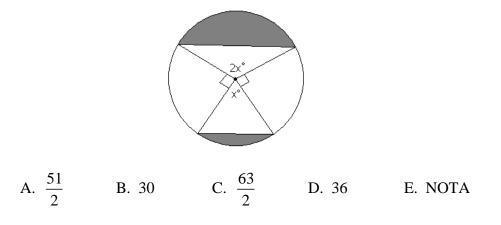
A. -5 B. -1 C. 1 D. $\frac{16}{3}$ E. NOTA

10. A tire with a radius of 2 feet travels a distance of 1 mile. How many complete revolutions does the tire make? (1 mile = 5280 feet)

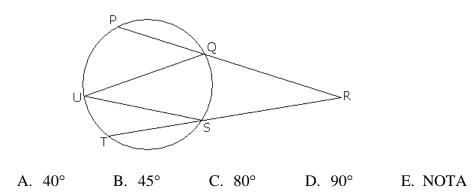
A.
$$\frac{1320}{\pi}$$
 B. $\frac{2640}{\pi}$ C. 1320π D. 2640π E. NOTA

- 11. A circle has center (0, 0) and radius 25. How many points on the circle have coordinates (x, y), where x and y are both integers?
 - A. 4 B. 8 C. 12 D. 20 E. NOTA

12. In the diagram below, the circumference of the circle is 12π . If the area of the shaded region is $P\pi - Q\sqrt{3}$, then what is the value of P + Q?



- 13. What is the shortest distance from the point (11, 13) to the circle with equation $x^2 + y^2 6x + 4y 36 = 0$?
 - A. 10 B. 11 C. 16 D. 17 E. NOTA
- 14. The points P, Q, S, T, and U lie on the circle shown. If $mPOT = 100^{\circ}$ and $m \angle R = 10^{\circ}$, then what is $m \angle U$?

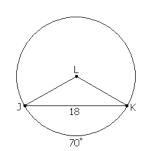


- 15. What is the measure of the obtuse angle formed by the hour hand and the minute hand of a clock at 9:24?
 - A. 126° B. 138° C. 162° D. 174° E. NOTA

16. How many points do the graphs of $x^2 + y^2 = 25$ and $y = x^2 - 1$ share?

A. 0 B. 2 C. 3 D. 4 E. NOTA

- 17. In how many different ways can 10 distinct keys be arranged on a keychain?
 - B. 9! C. $\frac{10!}{2}$ D. 10! A. $\frac{9!}{2}$ E. NOTA
- 18. Three distinct circles with centers P, Q, and R are tangent to each other. PQ = 8, QR = 10, and PR = 12. If r_1 , r_2 , and r_3 are the lengths of the radii of the three circles, then what is the value of the product $(r_1) (r_2) (r_3)$?
 - C. 120 A. 63 B. 105 D. 192 E. NOTA
- 19. A 72° sector of a circle has a radius of 10. The sector is made into a cone by connecting the two radii to each other. What is the height of this cone?
- C. $2\sqrt{26}$ A. $2\sqrt{21}$ B. $4\sqrt{6}$ D. $2\sqrt{58}$ E. NOTA 20. Which expression represents the area of circle L shown in the diagram below, given that JK = 18 and $m \mathcal{F} K = 70^{\circ}$?

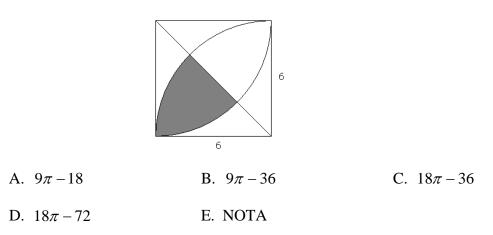


- B. $\frac{81\pi}{(\cos 35^{\circ})^2}$ C. $\frac{81\pi}{(\sin 70^{\circ})^2}$ A. $\frac{81\pi}{(\sin 35^{\circ})^2}$ D. $\frac{81\pi}{(\cos 70^{\circ})^2}$ E. NOTA
- 21. A rectangle with vertices (2, 1), (8, 1), (2, 4), and (8, 4) is revolved about the x-axis, creating a hollowed-out cylinder. What is the volume of this resulting solid?
 - B. 90*π* C. 96π D. 180π A. 54π E. NOTA
- 22. Circle O has diameter \overline{YZ} , and \overline{XY} is a tangent line to circle O. As point P begins at Y and then constantly moves closer to point X along XY (but point P never reaches point X), then what is the effect on $m \angle XPZ$?
 - B. it always increases A. it always decreases C. it remains the same
 - E. NOTA

D. it increases then decreases

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- 23. Mike and Wendy are standing on a merry-go-round which is making 3 revolutions per minute. Mike is standing 6 feet from the merry-go-round's center, and Wendy is standing 9 feet from the center. What is the positive difference in the distances traveled (in feet) by Mike and Wendy in one minute?
 - A. 0 B. 9π C. 18π D. 54π E. NOTA
- 24. The square in the diagram below has a side length of 6, and the arcs are circular arcs. The endpoint of each arc coincides with a vertex of the square. What is the area of the shaded region?



25. The eccentricity of a circle is

A. equal to 0	B. between 0 and 1	C. equal to 1
D. greater than 1	E. NOTA	

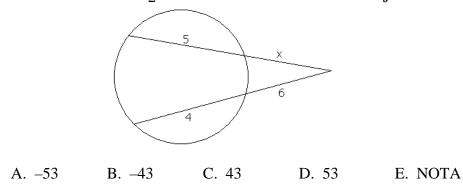
26. Circle P with radius 5 and circle Q with radius 8 are externally tangent to each other at point T. What is the length of a common external tangent segment to both circles which does not pass through T?

A. 12 B. $4\sqrt{10}$ C. $\sqrt{178}$ D. $\sqrt{194}$ E. NOTA

27. Three families of three people each, the Washingtons, the Jeffersons, and the Jacksons, are sitting down to dinner together at a round table with 9 seats. In how many different ways can the families sit so that each person is seated exactly 3 seats away from each other person in his family?

A. $9 \cdot 6 \cdot 3 \cdot 2 \cdot 2 \cdot 2$ B. $9 \cdot 9 \cdot 6 \cdot 3 \cdot 2 \cdot 2 \cdot 2$ C. 9!D. $9 \bullet 9!$ E. NOTA

- 28. Three concentric circles with radii 1, 3, and 6 make up a target. The target is laying flat on the ground. If a flat circular disk with radius 1 is tossed onto the target so that the disk lands at a random point, but some part of the disk lands inside the target, what is the probability that the disk lands entirely between the target's two largest circles?
 - A. $\frac{9}{49}$ B. $\frac{1}{4}$ C. $\frac{27}{49}$ D. $\frac{3}{4}$ E. NOTA
- 29. In the circle below, $x = \frac{1}{2} (J + \sqrt{K})$. What is the value of $-\frac{K}{I}$?



30. A salt shaker is in the shape of the cone frustum shown below. If a grain of salt has a volume of π/6, then what is the maximum number of grains of salt which can fit inside the shaker? Assume that the grains of salt fill the frustum with no space between them. (The height of the frustum is 2, and the radii of the bases are 6 and 10)

