

THETA GEMINI
2007 Mu Alpha Theta National Convention

The use of a calculator is **not** permitted.

For all questions, choice **E** is **NOTA**, meaning
“None Of These Answers.”

For this entire test, i is defined as $\sqrt{-1}$.

Diagrams are not necessarily drawn to scale.

1. Let $x = |2 - 2i|$, $y = \ln(\pi)$, and $z = (x + y)^0$.
Which of the following inequalities is true?
A) $x < y < z$ B) $x < z < y$ C) $y < z < x$
D) $z < y < x$ E) NOTA
2. The lines $x = -4$, $y = 2$, and $y = 2x$ form the borders of a triangular region in the Cartesian plane. What percent of the triangle's area lies in Quadrant III?
A) 32% B) 36% C) 64% D) 72% E) NOTA
3. Mr. Strauss draws two circles and three lines in a plane. He decides to mark with a purple dot every point at which two or more of these 5 figures intersect. At most, how many purple dots will he need?
A) 10 B) 11 C) 14 D) 17 E) NOTA
4. The only distinct roots of a polynomial function are -2 , 1 , and 3 . The function's y -intercept is 12 . What is the sum of the coefficients (including the constant term) of the polynomial?
A) -5 B) -2 C) 2 D) 24 E) NOTA

HOW TO PLAY TIC-TAC-TOE: The first player to claim all three squares in any given row, column, or diagonal is the winner. If no player does this, then there is no winner. Players alternate taking turns. On a player's turn, that player uses his/her letter to claim a square.

5. In a current game of Tic-Tac-Toe, shown below, **Player O** has just made her third move, and it is **Player X**'s turn.

If both players make any and all remaining turns by claiming a square selected completely at random from the available squares, then what is the probability that this game will **not** have a winner?

- A) $\frac{1}{2}$ B) $\frac{2}{3}$
C) $\frac{7}{9}$ D) $\frac{5}{6}$

E) NOTA

		O
	O	X
X	O	X

6. The graph of a conic section is centered at the point $(-1, 1)$ and has a vertex at the point $(-4, 1)$. The conic section has eccentricity $= 2$. Which of the following is true of the graph?
A) Its conjugate axis has length 10 .
B) Its conjugate axis has length $6\sqrt{3}$.
C) Its transverse axis has length 10 .
D) Its transverse axis has length $6\sqrt{3}$.
E) NOTA
7. Let $x = 1,401,010,112,010,410,020,410$. How many elements of the set $\{8, 9, 12, 15\}$ are factors of x ?
A) 1 B) 2 C) 3 D) 4 E) NOTA

8. If (a, b, c) is the solution to the system shown, then evaluate $(a+b+c)$.
- $$\begin{cases} x+3y+3z=5 \\ 2x-4y=6 \\ x+5y+z=1 \end{cases}$$

A) -1.5 B) -1 C) 0 D) 3 E) NOTA

9. The legs of a right triangle measure x and $2x$, and its hypotenuse measures 15. If $x = a\sqrt{b}$ in simplest radical form, then evaluate $|2a - b|$.

A) 1 B) 3 C) 5 D) 7 E) NOTA

10. Find the **units digit** of x , if $0.71\overline{46} = \frac{x}{9900}$.

A) 1 B) 3 C) 5 D) 7 E) NOTA

11. Emily, Pedro, and Nico invest money as a group and agree to share the profit equally. Emily contributes \$450, Pedro contributes \$300, and Nico contributes \$250. The investment yields a \$300 profit. How much more would Emily's share of the profit be if the profit were split proportionally based on each student's contribution to the initial investment?

A) \$25 B) \$30 C) \$40 D) \$50 E) NOTA

12. Let x be the next term of the sequence

$$\frac{5}{12}, \frac{17}{60}, \frac{13}{60}, \frac{37}{210}, \frac{25}{168}, \frac{65}{504}, \dots$$

If x can be expressed as $A^{-1} - B^{-1} + C^{-1}$, where A , B , and C are positive integers and $A < B < C$, then evaluate $|A + B - C|$.

A) 0 B) 3 C) 4 D) 7 E) NOTA

13. If $2 + 3\sqrt{2 + 3\sqrt{2 + 3\sqrt{2 + \dots}}}$ can be expressed in the form $\frac{a+b\sqrt{c}}{2}$, where c is prime and a and b are integers, then evaluate $|a + 2b - c|$.

A) 2 B) 7 C) 10 D) 24 E) NOTA

14. The clumsy cartoon bird DukDuk Goose utters expletives whenever he hurts himself. Of course, since the comics are family-friendly, each expletive is a series of distinct characters in a specific order. All characters are selected from the five-element set $\{ \$, \#, @, !, ? \}$.

If every expletive is at least three characters in length, then DukDuk has X distinct expletives available to his fowl mouth. Find the **sum of the digits** of X .

A) 3 B) 6 C) 8 D) 11 E) NOTA

15. The solutions to the equation $4x^3 - 7x + 3 = 0$ are the real numbers m , p , and t , where $m \leq p \leq t$. Evaluate $|3m - 5p + 3t|$.

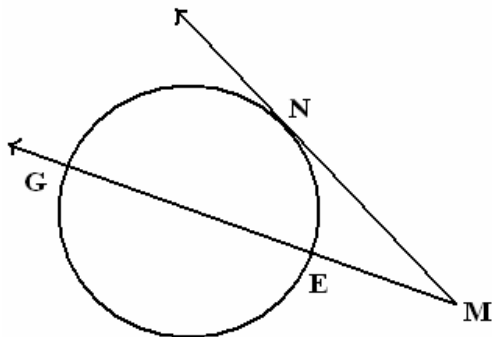
A) 0 B) 2 C) 4 D) 6 E) NOTA

16. Omar, Andrew, and Evan are playing a game that exactly one of them will win. The odds that Omar will win are 3:7. The odds that Andrew will win are 1:3. The odds that Evan will win are $A:B$, where A and B are relatively prime integers. Find $|B - A|$.

A) 1 B) 2 C) 11 D) 16 E) NOTA

17. \overline{MN} is tangent to the circle shown below at point N , and \overline{MG} intersects the circle at points E and G . The circumference of the circle is 32π , and the length of minor arc \widehat{EN} is 4π . The measure of minor arc \widehat{GN} is 100° , and $m\angle NME = x^\circ$. Find the **units digit** of x .

- A) 2
B) 4
C) 6
D) 8
E) NOTA



18. Evaluate: $(\sqrt{-2})(\sqrt{-8})$

- A) $-4i$ B) -4 C) $4i$ D) 4 E) NOTA

19. In the convex pentagon $ABCDE$, $m\angle A = (6x + 35)^\circ$, $m\angle B = (8x + 8)^\circ$, $m\angle C = (20x + 14)^\circ$, $m\angle D = 90^\circ$, and $m\angle E = (20x - 12)^\circ$. Find the degree measure of the largest acute angle in the pentagon.

- A) $77.\bar{3}$ B) 80 C) $81.\bar{3}$ D) $86.\bar{6}$ E) NOTA

20. A line perpendicular to the line $3x - 4y = 12$ passes through the points $(0, 2k + 1)$ and $(3 - k, 0)$. If k can be expressed as $\frac{P}{Q}$, where P and Q are relatively prime integers, then evaluate $|P - Q|$.

- A) 1 B) 6 C) 17 D) 18 E) NOTA

21. Find the sum of the real solutions to the equation: $\log(x - 1) + \log(x + 2) = 1$.

- A) -1 B) 2 C) 3 D) 6 E) NOTA

22. A one-to-one function is graphed in the Cartesian plane. Its domain is all real numbers on the closed interval $[-2, 5]$, and its range is all real numbers in the closed interval $[3, 10]$.

The graph is reflected about the line $y = x$, and then reflected again about the x -axis.

Let g be the function defined by the resultant graph. Which of the following statements are true?

- I. The range of g includes -1 .
II. The domain of g includes 3.
III. The range of g includes 3.

- A) I and II only B) I and III only
C) II and III only D) I, II, and III E) NOTA

23. For positive x , let f be a function such that $f(\sqrt{x} - 1) = \frac{7x + 5}{x}$. Find $f(4)$.

- A) 7.2 B) 8.25 C) $8.\bar{6}$ D) 12 E) NOTA

24. Rahn, Jorge, Thierry, Will, and Caitlin will enter the MAO Hall of Fame one at a time. Jorge must not enter first, and Thierry must be one of the last three to enter. In how many different orders can these five MAO alumni enter?

- A) 36 B) 54 C) 60 D) 72 E) NOTA

25. Let C be the set of all complex numbers that have an absolute value less than 3. Let Q be the set of all complex numbers in the form $a + bi$ where a and b are each real numbers that lie on the interval $[0, 3]$.

Find the probability that a number selected at random from set Q is also in set C . Round to the nearest tenth.

- A) 0.5 B) 0.7 C) 0.8 D) 1.0 E) NOTA

26. Consider rhombus $ABCD$. If $BD + AC = 32$, $(BC)^2 = k$, and the area of the rhombus is $(k - 4)$, then find the **tens digit** of k .

- A) 1 B) 2 C) 3 D) 6 E) NOTA

27. Express the base-5 number $(1032)_5$ in base-8.

- A) $(162)_8$ B) $(216)_8$ C) $(217)_8$
 D) $(271)_8$ E) NOTA

28. A regular hexagon's apothem measures $\sqrt{5}$. The hexagon is the base of a right prism. The prism's height is $2\sqrt{6}$. The volume of the prism is $x\sqrt{y}$ in simplest radical form. Find the **sum of the digits** of the product (xy) .

- A) 3 B) 6 C) 9 D) 11 E) NOTA

29. A vat contains 80 gallons of liquid that is 35% rubbing alcohol. How many gallons of water must be added to the vat so that the liquid in the vat is 20% rubbing alcohol?

- A) 15 B) 24 C) 32 D) 60 E) NOTA

30. A *lattice point* is a point (x, y) where x and y are both integers. If there are L lattice points that lie in the solution set of the system below, then find the **sum of the digits** of L .

$$\begin{cases} 16x^2 + 9y^2 - 64x + 18y \leq 71 \\ y \geq \frac{1}{2}|x - 2| - 1 \end{cases}$$

- A) 4 B) 6 C) 8 D) 10 E) NOTA

*This grid is provided for your convenience
 by the Florida Board of Tourism.*

