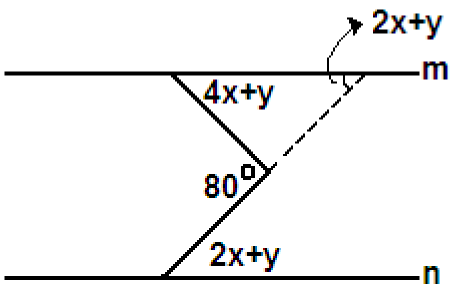
Theta Geometry Solutions 2007 Mu Alpha Theta National Convention

1. A:
Box has dimensions x = 5 cm, y = 10 cm, and z = unknown
$Surface\ \ Area = Perimeter_{base} \times Height + 2Base_{area} = (2\cdot 5 + 2\cdot 10)z + 2\cdot 5\cdot 10 = 250$
Solving for z, gives $z = 5\,cm$
$Volume = 5\cdot 5\cdot 10 = 250\,cm^3$

2. D:
By alternate interior angles:



Therefore: $2x + y + 4x + y + (180° - 80°) = 180°$
$6x + 2y = 80° \rightarrow 3x + y = 40°$

3. D:
Angle formed by hands of a clock = $|30h - 5.5m| = |30\cdot 2 - 5.5\cdot 22| = 61°$

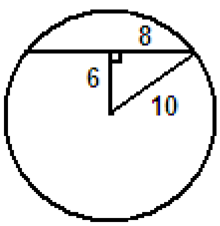
4. D:
Perpendicular line has slope 5. → $y = 5x + b$ → $22 = 5\cdot 2 + b$ → b = 12

5. A:
The rope is composed of the two halves of the pulleys + two straight pieces connecting them.
$\frac{20\pi}{2} + \frac{20\pi}{2} + 80 + 80 = 160 + 20\pi$

6. C:
Squares enclosed = $8 \cdot 8 + 7 \cdot 7 + 6 \cdot 6 + 5 \cdot 5 + 4 \cdot 4 + 3 \cdot 3 + 2 \cdot 2 + 1 = 204$

7. D:
Let length $AE = x + 3$ → $4(4+8) = 3(3+x) \rightarrow x = 13$ → $AE = 16$
Measure of $\angle A = \frac{(CE - BE)}{2} = \frac{(80-20)}{2} = 30°$
30+16=46

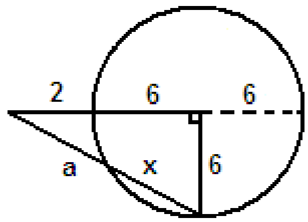
8. C:



9. B:
A→ $90 - x = \frac{1}{3}(180 - x) \rightarrow x = 45°$
B→ $\sqrt{3 \cdot 27} = 9$
(A+B)/2 = 27

10. B:
x=(-1+5+8)/3=4
y=(2+10-3)/3=3
(x,y)=(4,3)

11. A:



Let x be the distance in the hypotenuse that lies inside the circle.

$(6+6+2)(2) = x(a+x) = 28$

$(a+x)^2 - 36 = 64 \rightarrow (a+x) = 10$

$28 = x(a+x) = x(10) \rightarrow x = \frac{14}{5}$

12. E. 3770

$\frac{n(n-3)}{2} + 180(n-2) + 360 = \frac{20(20-3)}{2} + 180(20-2) + 360 = 3770$

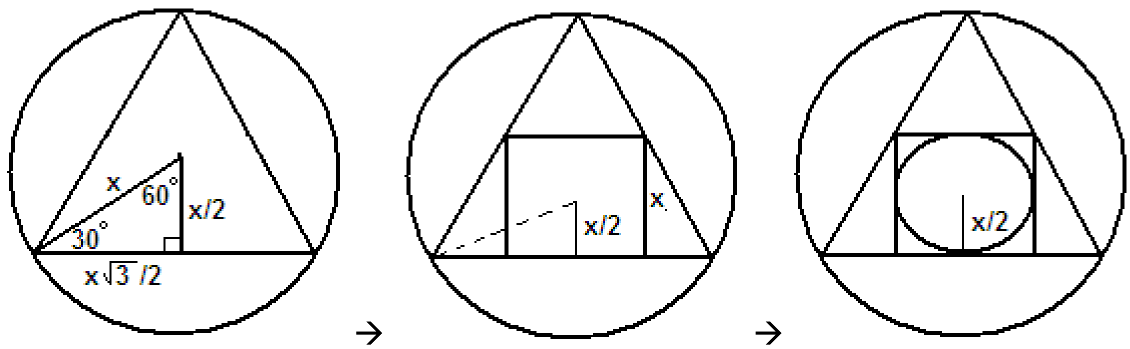
13. B:

$V_{original} = x^2 \cdot \pi \cdot \frac{h}{3}$

$V_{new} = y^2 \cdot \pi \cdot \frac{h}{3}$

$V_{new} = 3V_{original} \rightarrow y^2 \pi \frac{h}{3} = x^2 \pi \cdot h \rightarrow \frac{y}{x} = \sqrt{3}$

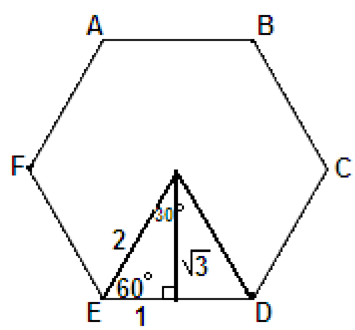
14. A:



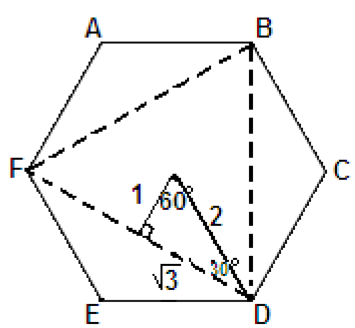
Area of remaining circle = $\frac{x^2}{4}\pi$ = ¼ of the original circle.

15. C:

Sum of triangles we are looking for = Area of the hexagon – Area of equilateral triangle FBD.



Area of ABCDEF = $\sqrt{3}\cdot 2\cdot \frac{1}{2}\cdot 6 = 6\sqrt{3}$



Area of FBD = $\frac{(2\sqrt{3})^2\sqrt{3}}{4} = 3\sqrt{3}$

FAB + BCD + FED = $6\sqrt{3} - 3\sqrt{3} = 3\sqrt{3}$

16. E. $\frac{\pi - 2}{\pi}$

Area of 1 segment = $x^2\pi \cdot \frac{90}{360} - \frac{x^2}{2} = \frac{x^2}{2}(\frac{\pi}{2} - 1)$

4 segments = $2x^2(\frac{\pi}{2} - 1)$

Area of pizza = $x^2\pi$

Ratio = $\frac{\pi - 2}{\pi}$

17. C:

Let x be the distance from point M to A, B and $\overline{CD}$

$(16 - x)^2 + 8^2 = x^2 \rightarrow x = 10$

$BAM = \frac{(16 - x)(16)}{2} = 48$

18. C:

Angle B = $\frac{180(8 - 2)}{8} = 135$

$\angle APC = 360 - 135 \cdot 2 = 90$

Area = $\frac{90}{360} \cdot 8^2\pi = 16\pi$

19. D:

Angle formed at point B by junction of octagon and triangle = 135+60=195

Other polygon must have angles of 360-195=165

$165 = \frac{180(n - 2)}{n} \rightarrow n = 24$

20. B:

Let x = radius of smaller circle→2x = smaller base of the trapezoid

Altitude = 10

10-x=radius of larger circle→20-2x = larger base of the trapezoid

Area = $\frac{1}{2}(2x + 20 - 2x)(10) = 100$

21. C:

$\frac{8}{x}=\frac{10}{12-x}\rightarrow x=\frac{16}{3}$

$DC=12-x=\frac{20}{3}$

22. D:

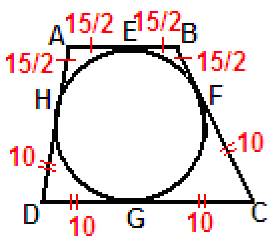
Given leg is the geometric mean of the segment adjacent to it and the hypotenuse:

$\frac{4}{6}=\frac{6}{BC}\rightarrow BC=9$

23. A:

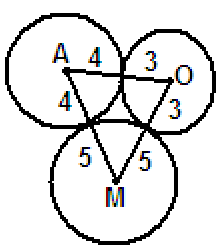
Tangents from the same external point are congruent, so:

$\overline{DH}=\overline{DG}, \overline{CG}=\overline{CF}, \overline{AH}=\overline{AE}, \overline{BF}=\overline{BE}$



Perimeter = $15\cdot 2+10\cdot 4=70$

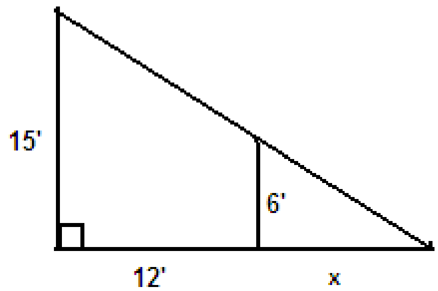
24. D:



$\overline{MA}=9, \overline{MO}=8, \overline{AO}=7$

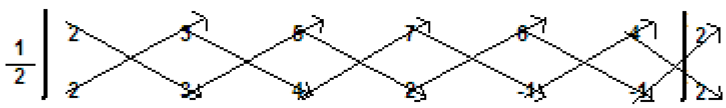
By Heron's Theorem: $Area=\sqrt{SP(SP-\overline{MA})(SP-\overline{MO})(SP-\overline{AO})}=\sqrt{12(3)(4)(5)}=12\sqrt{5}$

25. B:



$$\frac{15}{12+x}=\frac{6}{x}\rightarrow x=8\rightarrow 12+8=20$$

26. E. 16

Area = 

*with vertices in clockwise order

$$=\frac{1}{2}(2\cdot 3+3\cdot 4+5\cdot 2+7\cdot(-1)+6\cdot(-1)+4\cdot 2-2\cdot 3-3\cdot 5-4\cdot 7-2\cdot 6-(-1)\cdot 4-(-1)\cdot 2)=16$$

27. C:

Height of parallelogram = $\sqrt{10^2-8^2}=6$

Area of parallelogram = 6(8) = 48

Area of square = 16(6) = 96

96-48 = 48

28. C:

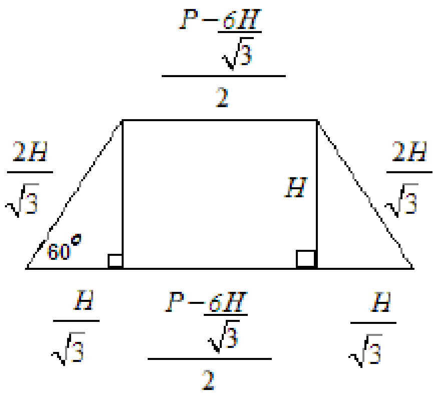
$\overline{AE}=24, \overline{AD}=40$

$\sqrt{\overline{AD}^2-\overline{AE}^2}=32$

$\overline{BD}=7+32=39$

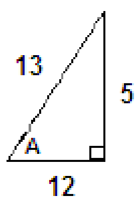
Area = $\frac{1}{2}diagonal\cdot diagonal=.5\cdot 48\cdot 39=936$

29. A:



$$A=\frac{1}{2}H\left(\frac{P}{2}-\frac{3H}{\sqrt{3}}+\frac{P}{2}-\frac{3H}{\sqrt{3}}+\frac{2H}{\sqrt{3}}\right)=\frac{H}{2}\left(P-\frac{4H}{\sqrt{3}}\right)=\sqrt{3}\left(14+2\sqrt{3}-\frac{4\cdot 2\sqrt{3}}{\sqrt{3}}\right)=6+6\sqrt{3}$$

30. B:



cos(A)= $\frac{adjacent}{hypotenuse}=\frac{12}{13}$