Theta Geometry Test
2007 Mu Alpha Theta National Convention
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For all questions, answer E. "NOTA" means none of the above answers is correct.

1. A right rectangular box has surface area $250\,cm^2$ and has a base width and length of $5\,cm$ and $10\,cm$, respectively. What is its volume?

    A. $250\,cm^3$ B. $125\,cm^3$

    C. $100\,cm^3$ D. $5\,cm^3$ E. NOTA

2. Lines **m** and **n** are parallel. Using the angle measures shown in the diagram below, find the value of 3x+y,
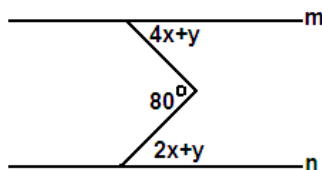
    A. 100°   B. 80°
    C. 50°   D. 40°   E. NOTA

3. What is the angle formed by the minute and hour hands of a clock when it is 2:22 pm?

    A. 22°   B. 30°
    C. 60°   D. 61°   E. NOTA

4. Find the y-intercept of the line perpendicular to $y = \dfrac{-x}{5} + 6$ passing through (2,22).

    A. $(0, -5)$ B. $(0, 2)$
    C. $(0, 5)$ D. $(0, 12)$ E. NOTA

5. Two congruent circular pulleys are 20 inches in diameter and their centers are 80 inches apart. How long, in inches, must an unbroken belt be to tightly wrap around both pulleys as shown below?

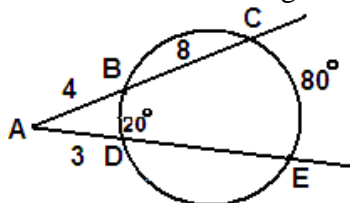
    A. $160 + 20\pi$ B. $120 + 20\pi$
    C. $80 + 20\pi$ D. $80 + 10\pi$ E. NOTA

6. How many squares are enclosed in a chess board made of 8 squares by 8 squares?

    A. 64   B. 128
    C. 204   D. 256   E. NOTA

7. In the figure below, arcs CE and BD measure $80°$ and $20°$, respectively. Segments AB, BC, AD measure 4, 8, and 3 cm respectively. If $\angle A$ measures k degrees and $AE = r$, find the value of $k + r$.

    A. 16   B. 30

    C. 33   D. 46   E. NOTA

8. A circle with diameter $20\,cm$ has a chord $16\,cm$ long. What is the distance between the chord and the center of the circle?

    A. $4\,cm$ B. $5\,cm$

    C. $6\,cm$ D. $7\,cm$ E. NOTA

9. Let A degrees be the measure of an acute angle whose complement measures 1/3 of its supplement. Let B equal the geometric mean of 3 and 27. Find the arithmetic mean of A and B.

    A. 20   B. 27

    C. 35   D. 45   E. NOTA

10. Triangle ABC has coordinates A(-1,2), B(5,10), C(8,-3). Find the coordinates of its centroid.

    A. (3,3)   B. (4,3)

    C. (4,4)   D. (5,4)   E. NOTA

11. In the shown diagram, the circle has radius 6 and segment measure **a** lies entirely outside the circle. Find **a**.
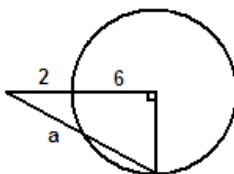
    A. $\dfrac{14}{5}$ B. $\dfrac{7}{2}$

    C. $\dfrac{9}{2}$ D. 6   E. NOTA

12. Find the sum of the number of diagonals and the number of degrees in the sum of the interior and exterior angles of a regular icosahedron. Note: an icosahedron is a polygon with 20 sides.

    A. 3760   B. 3940

    C. 3960   D. 3970   E. NOTA

13. The volume of a right circular cone is tripled, with its height remaining constant. What is the ratio of the radius of the new cone to the radius of the original cone?

A. $\dfrac{\sqrt{3}}{3}$

B. $\sqrt{3}$

C. 3

D. $3\sqrt{3}$

E. NOTA

14. Starting with a circular piece of paper, I trim the paper so as to leave the largest possible equilateral triangular piece of paper. Starting again, I trim the equilateral triangular piece of paper so as to leave the largest possible square piece of paper. Starting one last time, I trim the square piece of paper so as to leave the largest possible circular piece of paper. What fraction of the original piece of paper remains after the last cut?

Note: The cuts are shown below represented by dashed lines.
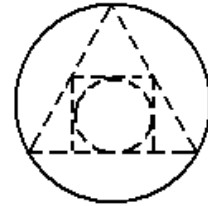
A. $\dfrac{1}{4}$

B. $\dfrac{1}{3}$

C. $\dfrac{1}{2}$

D. $\dfrac{2}{3}$

E. NOTA

15. Given regular polygon ABCDEF with side length 2, find the sum of the areas of triangles FAB, BCD, and FED.

A. $\dfrac{1}{4}$

B. $\dfrac{3\sqrt{3}}{2}$

C. $3\sqrt{3}$

D. $5\sqrt{3}$

E. NOTA

16. Yan is hungry and buys a small circular pizza which comes pre-cut into 4 slices. The pizza has radius **x** cm. However, Yan does not like to eat the crust and throws away a segment from each slice containing the crust. What fraction of the original pizza does Yan end up throwing away?

A. $\dfrac{1-\pi}{\pi}$

B. $\dfrac{2-\pi}{\pi}$

C. $\dfrac{\pi-1}{2\pi}$

D. $\dfrac{2}{\pi}$

E. NOTA

17. In square ABCD, a point M lies in its interior whose distance to A, B and $\overline{CD}$ are equal. Find the area of BAM if CD = 16.

A. 24

B. 32

C. 48

D. 64

E. NOTA

18. A regular octagon ABCDEFGH is inscribed in circle P of radius 8. Find the area of the sector formed by the radii of the circle and arc ABC.

    A. 8π B. 12π

    C. 16π D. 20π E. NOTA

19. Regular octagon ABCDEFGH and equilateral triangle PBC have side BC in common (with point P being on the exterior of the octagon). $\angle PBA$ is one of another regular polygon's interior angles. How many sides does this polygon have?

    A. 6   B. 12

    C. 20   D. 24   E. NOTA

20. Two non-congruent circles are externally tangent. Each base of an isosceles trapezoid is a diameter of one of the circles. If the distance between the centers of the circles is 10, what is the area of the trapezoid?
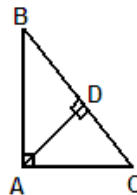
    A. 50   B. 100

    C. 128   D. 256   E. NOTA

21. In triangle ABC, segment $\overline{BD}$ bisects $\angle ABC$, with point D lying on $\overline{AC}$. If $AB = 8$, $BC = 10$, $AC = 12$, find DC.

    A. 4   B. $\dfrac{16}{3}$

    C. $\dfrac{20}{3}$ D. $\dfrac{25}{3}$ E. NOTA

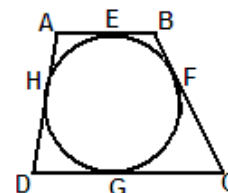22. If $AB = 6$, $BD = 4$, find BC.

    A. 36   B. 24

    C. 12   D. 9   E. NOTA

23. Circle O is inscribed in quadrilateral ABCD. The points of tangency are E, F, G and H. If $AH = BF$, $DG = FC$, $DH = 10$, and $AB = 15$, find the perimeter of ABCD.

    A. 70   B. 60

    C. 50   D. 40   E. NOTA

24. Circles M, A, and O are externally tangent to each other. Their diameters have lengths 10, 8, and 6 respectively. Find the area of triangle MAO.

    A. 6   B. $6\sqrt{3}$

    C. 24   D. $12\sqrt{5}$ E. NOTA

25. On a sunny day a man stands along the shadow of a tree at a distance of 12 ft from its trunk. The man and the tree are 6 ft and 15 ft tall respectively. Find the length of the shadow of the tree. (neglect the thickness of the tree trunk)
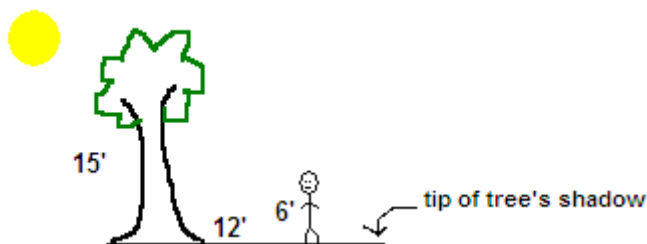
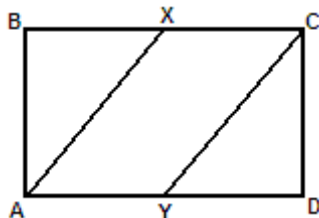   A. 25 ft   B. 20 ft

    C. 16 ft   D. 8 ft   E. NOTA

26. A convex hexagon has vertices at (3,3), (2,2), (7,2), (5,4), (4, -1) and (6,-1). Find its area.

    A. $\dfrac{9}{2}$ B. $\dfrac{11}{2}$

    C. $\dfrac{27}{2}$ D. $\dfrac{81}{2}$ E. NOTA

27. A parallelogram is inscribed in a rectangle as shown, with two of its parallel sides lying on the rectangle. If $BX = 8$, $CY = 10$ and $\overline{XY} \parallel \overline{AB}$, find the area inside the rectangle that is outside of the parallelogram.

    A. 24   B. 32
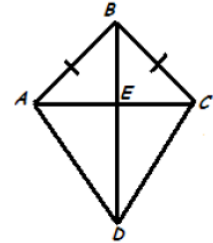
    C. 48   D. 64   E. NOTA

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28. Kite ABCD is shown below. Its diagonals intersect at point E. If $\overline{AC} = 48$, $\overline{BE} = 7$, and $\overline{AD} = 40$, what is the kite's area?

    A. 1872   B. 1536

    C. 936   D. 768   E. NOTA

29. An isosceles trapezoid has a height of $2\sqrt{3}$, perimeter of $14 + 2\sqrt{3}$ and a base angle $60°$. Find its area.

    A. $6 + 6\sqrt{3}$ B. $12 + 12\sqrt{3}$

    C. $24 + 12\sqrt{3}$ D. $12 + 24\sqrt{3}$ E. NOTA

30. Given $\sin(A) = \dfrac{5}{13}$, find $\cos(A)$, where A is an acute angle.

    A. $\dfrac{5}{13}$ B. $\dfrac{12}{13}$

    C. $\dfrac{13}{12}$ D. $\dfrac{13}{5}$ E. NOTA