1) B ($2\sqrt{3}$)

Using 30-60-90 and 45-45-90 triangles, we can see that AC $= 2 + 2\sqrt{3}$ and that BC $= 2\sqrt{3}$.

2) A ($-2+\sqrt{6}$)

We can set the original expression equal to x and then note that $x = \dfrac{1}{1+\frac{3}{2+x}}$. Rationalizing the denominator gets $\dfrac{2+x}{5+x} = x$. Solving for x and removing the extraneous solution, $x = -2+\sqrt{6}$

3)C (147π)

Volume $= *frac(R^2+Rr+r^2)\pi 3, R = 6, r = 3, h = 7$. Thus, V $= 147\pi$ (one could have used similar triangles and subtracted the small cone from the big cone).

4) B ($\frac{1}{3}$)

It might seem obvious at first glance that you have a 50-50 shot a winning, but let's look closer. If you decide to stay, for you to win, you must have originally picked the correct door. Thus the probability is $\frac{1}{3}$. Just for fun, if you had decided to always switch, you would win if you originally picked the wrong door (you originally pick the wrong door then switch to the right one). The probability of that happening is $\frac{2}{3}$ and thus, switching is actually the correct strategy.

5)B (16)

The ellipse can be written as $\dfrac{(x-1)^2}{25} + \dfrac{(y-2)^2}{9} = 1$ and thus, major axis = 10, and minor axis = 6. 10 + 6 = 16

6) D (72_9)

$72_9 = 7(9) + 2(1) = 65$
$2012_3 = 2(27) + 1(9) + 0(3) + 2(1) = 65$ So, $72_9 = 2012_3$.

7) D (112)

The constant term will be $(85)(4x^5)^3(.5x^{-3})^5 = 56(64)(1/32) = 112$.

8) B (5,3)

To find the centroid we sum all the x values and sum all the y values and then divide by 3. Thus, (5,3).

9) E (IV)

Only IV has these properties.

10) D ($x < -3$ or $1 < x \leq 5$)

If I call values of x for which one or more solutions are 0 or undefined 'critical values of x' then, we must find the critical points. The first is found by setting the expressions equal to each other. This leads to $x = 5$. The other two are the points where the expressions are undefined. Thus, $x = -3, 1$ are also critical points. We then check where the inequality is true. We see its true for $x < -3$ or $1 < x \leq 5$.

11) D ($\frac{3}{4}$)

Let $S = \frac{1}{3} + \frac{2}{9} + \frac{3}{27} + \frac{4}{81}...$ Then
$3S = 1 + \frac{2}{3} + \frac{3}{9} + \frac{4}{27}...$
$3S - S = (\frac{1}{3} + \frac{2}{9} + \frac{3}{27} + \frac{4}{81}...) - (1 + \frac{2}{3} + \frac{3}{9} + \frac{4}{27}...) = 1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27}... = \frac{3}{2} = 2S$
Thus, $S = \frac{3}{4}$

12) E (0)

The sum of the roots is -2(0)/3 = 0

13) C (8)

The length of the latus rectum is 4 times the distance between the directrix and the vertex. So 4*2 = 8.

14) B (1)

Simplifying the log equations leads to $\frac{100x^2}{(x-1)^4} = 100$. Taking the square roots of both sides lead to the two quadratic equations: $x = (x-1)^2$ and $x = -(x-1)^2$. These are equal to $x^2 - 3x + 1 = 0$ and $x^2 - x + 1 = 0$. The first equation only has one solution that is not extraneous and the second equation has none. Therefore, there is only 1 solution.

15) E (0)

Factoring the expression results in $(n-4)(n+1)(n^2+7)$. Thus, $i = 4$ is a root. When the product gets to $n = 4$, there will be a 0 causing the entire product to be 0.

16) B (1)

The rational function factors to $f(x) = \frac{(x-1)(x-5)(x-7)}{(x-1)(x-8)(x^2+4)}$. Thus, there is 1 removable discontinuity.

17) B (20)

$5!/3! = 20$

18) C (336)

If A and B are square matricies, $det(AB) = (detA)(detB)$. Thus, we just need to find the determinant of each matrix and then multiply them. This first matrix has a determinant of 24 and the second one is 14. Therefore, the determinant of A is 336.

19) B (5)

The radius is nothing more than the shortest distance from the center to the tangent line. Thus, we can use the point-to-a-line formula. $\frac{\|(3)(3) + (4)(2) + (-42)\|}{3^2 + (4)^2} = 5$.

20) B (195 m)

Let the three unique side lengths of the prism to be a, b, and c. The surface area information tells us that $2ab + 2ac + 2bc = 334$. The edge length information tell us $4a + 4b + 4c = 92$. Thus, $ab + ac + bc = 167$ and $a + b + c = 23$ and we are looking for $(\sqrt{a^2 + b^2 + c^2})^2$ and therefore want to find $a^2 + b^2 + c^2$. But that is the sum of the squares of the roots of a cubic equation with roots a, b, and c. We also have the sum of the roots and the sum of the roots taken two at a time of this cubic equation. Therefore, we can use the formula that relates the three ((sum of the square of the roots) = (sum of the roots squared) - 2(sum of the roots taken two at a time)). Thus $a^2 + b^2 + c^2 = 23^2 - 2(167) = 195$

21) D (8)

First, we will write the general equation of the line with slope m that goes through the point (4,16): $y - 16 = m(x - 4) \rightarrow y = mx + (16 - 4m)$. Now, when we set the equation of a line and a parabola equal to each other, the roots of the resulting equation tell us the x values for which the line and the parabola intersect. Since this line is tangent to the parabola, we want to make it so that equation only has one solution. Since the resulting equation is quadratic, we can do this by setting the discrimant equal to zero. So, we have $y = mx + (16 - 4m)$ and $y = x^2 \rightarrow x^2 = mx + (16 - 4m) \rightarrow x^2 - mx + (4m - 16) = 0$. Now, setting the discrimant equal to zero leads us to $m^2 - 4(4m - 16) = 0 \rightarrow m^2 - 16m + 64 \rightarrow (m - 8)^2 = 0$ and thus $m = 8$. Now since m is the slope of the tangent line at (4,16), we have our answer.

22) D (801)

We can find the max possible value for S and then the min value for S. It is clear that all numbers between the max and min are possible values for S. Max S $= 80(\dfrac{90+11}{2}) = 4040$. Min S $= 80(\dfrac{80+1}{2}) = 3240$. Thus, the amount of values between 4040 and 3240 inclusive is $4040 - 3240 + 1 = 801$.

23) A ($\frac{1}{2}$)

Consider the situation after Ann and Mary have each flipped the coin 10 times. There are 3 possibilities:

1. Ann has more heads than Mary.
2. Mary has more heads than Ann.
3. Ann and Mary have an equal number of heads.

Ann will always win in case 1, and Mary will always win in case 2. These two cases are equally likely. In case 3 Mary wins if her 11th flip is heads, and loses otherwise. Each of these outcomes is equally likely. Hence Ann and Mary have an equal chance of winning, i.e., $\frac{1}{2}$.

24) A (10)

Using the p/q list and synthetic division, we get the roots to be -1,3,4, and 7. Thus, -1 + 4 +7 = 10.

25) C (50)

This is just (5 C 4) * (5 C 3) = 50

26) D (none)

If ϵ is associative, then $(a \ \epsilon \ b) \ \epsilon \ c = a \ \epsilon \ (b \ \epsilon \ c)$.
Thus, $(ab + a - b)c + ab + a - b - c \ ?=? \ a(bc + b - c) + a - (bc + b - c)$
$\rightarrow abc + ac - bc + ab + a - b - c \ ?=? \ abc + ab - ac + a - bc - b + c$. This clearly is false, so ϵ is not associative.

If If ι is commutative, then $a \ \iota \ b = b \ \iota \ a \rightarrow a - b \ ?=? \ b - a$. This is also clearly false, so ι is not commutative.

27) B $(x^2 - 4x - 17)$

$j(x) = x^2 - 10x + 4 \rightarrow j(x+3) = (x+3)^2 - 10(x+3) + 4 = x^2 + 6x + 9 - 10x - 30 + 4 = x^2 - 4x - 17$

28) C$(-4 - 4i)$

This equals $(2i)^2 * (1+i) = -4-4i$

29) C (54)

$8820 = (2^2)(3^2)(5^1)(7^2)$. Using the well known trick, we get $(2+1)(2+1)(1+1)(2+1) = 54$.

30) C (37)

$3^2 - 4(-7) = 37$