

## Logs, Exponents/Radicals

### Solutions

#### Solutions:

1. **A.**  $2q^{3(x+2)} = 2^{4x}$  so exponents are equal:

$3(x+2)+1=4x$  and  $x=7$  and  $10-x=3$ .

2. **A.**  $\frac{3^{4x} + 3^{4x}}{3^{-x}} = 6$  so

$(2 \cdot 3^{4x}) \cdot 3^x = 6$  and so

the 3 exponent must be equal to 1.

So  $x=1/5$ .

3. **B.** Move terms to the left and factor by

grouping:  $x^{\frac{2}{3}}(x^{\frac{1}{3}} + 4) - 1(x^{\frac{1}{3}} + 4) = 0$

$(x^{\frac{1}{3}} + 4)(x^{\frac{2}{3}} - 1) = 0$  gives  $q, r, s$  are 1

and  $-1$ , and  $-64$  and  $|qrs/8|=8$ .

4. **D.** Divide each side by  $x$ , and by 3 and

get  $\frac{2}{3} = x^{\frac{1}{3}}$  and then cube both

sides to

get  $x=8/27$ . So  $a+2b=8+54=62$ .

5. **E.**  $\sqrt{3} + 3 + \sqrt{27} = 4\sqrt{3} + 3$  so  $p+q=7$ .

6. **C.** Square the first equation to get

$x + 2\sqrt{xy} + y = 36$  and using the second

equation gives  $x+y=36-4=32$ .

7. **C.** Cube root both sides to get

$$\frac{x+1}{x-1} = \frac{1}{2}$$

## 2007 Mu Alpha Theta National Convention

which solves to  $x = -3$ , so

$|x|+1=4$ .

8. **D.** The domain of  $f$  is  $[1, \infty)$  and the

domain of  $g$  is  $(-\infty, 1]$  so  $a$  and  $b$  are

both 1.

9. **B.** Square both sides to get

$x - \sqrt[3]{2 - (4)} = 16$ . The inner parentheses

comes from substitution of the original

equation which was equal to 4.

So

$$x = 16 - \sqrt[3]{2} = 2^4 - 2^{\frac{1}{3}} \text{ so}$$

$6(4)(1/3)=8$ .

10. **C.** Two values:  $x=1$  and  $x=4$ .

11. **B.** The inverse of  $y = x^{\frac{2}{5}}$  is

$y = x^{\frac{5}{2}}$  for

positive values of  $x$ .  $g(2) = \sqrt{32}$ .

12. **D.**  $g(x) = \sqrt{x}$  and  $\sqrt{x} > 20$  gives

$x > 400$  but since  $g(x)$  must be an

integer then  $x$  must be 21 squared, or 441.

13. **B.** The sum and difference of cubes

formulas give  $x^3 + 8 = 9$  and

$y^3 - 27 = 10$  so  $x^3 = 1$ ,  $y^3 = 37$

and the

product  $(xy)^3 = 37$ .

Logs, Exponents/Radicals

Solutions

14. **D.** The expression simplifies

$$\text{to } \frac{1}{4}x^{-\frac{13}{2}}$$

so  $4\left(\frac{1}{4}\right)\left(-\frac{13}{2}\right)$  gives  $-13/2$ .

15. **B.**  $a^{\left(\frac{-1}{3} + \frac{-1}{2} + 2\right)} = a^{\frac{1}{6}}$  so  $a$  is a perfect sixth

power, the least of which is  $2^6$

16. **D.**  $f(3) = (8)^{\frac{2}{3}} = 4$  and

$$f(2) = 8^{\frac{1}{2}} = 2\sqrt{2}.$$

17. **C.**  $(2)^{(-3)(-3)} = 512$

18. **B.**  $(x+1)^{\frac{m}{17}} = 36$  and

$$x+1 = 36^{\frac{17}{m}}, \text{ and}$$

the values of  $m$  which give an integer

answer are 2 or 17, since 36 is a perfect

square.  $x = 36^{\frac{17}{2}} - 1 = 6^{17} - 1$ . And  $k=17$ .

19. **B.**  $4^{\frac{1}{4}} = 2^{\frac{2}{4}} = 2^{\frac{1}{2}}$

20. **E.**  $(12^4 - 1) = (12^2 - 1)(12^2 + 1)$

so 143

and 145 are both factors. The first

factor can be reduced to  $(12-1)(12+1)$  so

11 and 13 are factors. All are therefore factors.

2007 Mu Alpha Theta National Convention

21. **B.** Square both sides to get

$$13 + 2\sqrt{22} = a + b + 2\sqrt{ab} \text{ so}$$

$ab=22$

and  $a+b=13$ . Integers 2 and 11 make

both equations true, so  $a-b=9$ .

22. **E.** Factor

$$x^3 + 7^3 = (x+7)(x^2 - 7x + 49)$$

and divide by  $x+7$ .

23. **B.**  $\sum_{x=1}^n x = \frac{1}{2}n(n+1)$  and

$$\sum_{x=1}^n x^3 = \left(\frac{1}{2}n(n+1)\right)^2 \text{ so } a-$$

$b=0$ .

If you do not know the formulas,

then see the pattern by using two

terms and then three terms.

Each time

the difference is 0.

$$24. \text{ **C.** } \binom{12}{0} + \binom{12}{1} + \dots + \binom{12}{12} = 2^{12}$$

as each

row of Pascal's triangle gives

$2^{\text{that row}}$  so

$b=12$ .

25. **A.** The equation gives

$$\log_2(\log_2 x) = 8$$

and then  $2^8 = \log_2 x$  and lastly

$$2^{2^8} = x$$

and the only prime factor is 2.

26. **B.**  $2^{\frac{x}{y}} = 2^{1+\frac{3}{x}}$  so  $\frac{x}{y} = 1 + \frac{3}{x}$  and

$$\frac{x}{y} = \frac{x+3}{x} \text{ which solves to}$$

$$\frac{x^2}{x+3}.$$

27. **D.**  $3^{2x} - 3^x + 2 = 0$  factors to

$(3^x - 2)(3^x - 1) = 0$ . Since  $x$  is not 0

we know  $3^x = 2$  so  $3^{-x} = \frac{1}{2}$

28. **A.**  $a = x^{-1}, b = y^{-1}$  and solve the system

$$a - b = 1 - 1/4, \quad 2a - 5b = -1/3,$$

to

get  $a = -11/36$  so  $x = -36/11$ .

Take the

absolute value and  $(36+2)/11$  is the

result.

29. **C.**

$$\frac{1}{1-i} + 1 + (1-i) + (-2i) + -2i(1-i)$$

which gives

$$\frac{1+i}{2} + 1 + 1 - i - 2i + -2i - 2$$

$$= \frac{1+i}{2} - 5i = \frac{1}{2} - \frac{9}{2}i.$$

30. **C.** From the set

$\{-4, -3, -1, 2, 3, 5\}$  we

have possible exponents :

$$\frac{-4}{-3}, \frac{-4}{-1}, \frac{-4}{2}, \frac{-4}{3}, \frac{-4}{5}$$