THETA Polynomials & Rational Functions SOLUTIONS 2007 Mu Alpha Theta National Convention

For all questions, answer E. "NOTA" means none of the above answers is correct.

Calculator use – NO calculators will be permitted on any test other than the Statistics topic test.

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1. C
2. B
3. E
4. A
5. C
6. D
7. E
8. B
9. A
10. D
11. A
12. E
13. B
14. D
15. B
16. D
17. C
18. D
19. A
20. D
21. B
22. B
23. B
24. C
25. C
26. A
27. C
28. D
29. A
30. B

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1. C:

$f(x) = x^3 + x^2 - x + 1 = (-1)^3 + (-1)^2 - (-1) + 1 = -1 + 1 + 1 + 1 = 2$

2. B:

Slope of the line = $\frac{-1-3}{-6-2} = \frac{1}{2}$

$y = mx + b \rightarrow 3 = \frac{1}{2} \cdot 2 + b \rightarrow b = 2$

$y = \frac{x}{2} + 2$

3. E: 1 vertical, 1 slant:

$f(x) = \frac{(x-1)(x+5)(x+2)}{(x-1)(x-5)} = \frac{(x+5)(x+2)}{x-5}$

Vertical asymptote → $x - 5 = 0 \Rightarrow x = 5$

Numerator with greater degree than denominator, so *slant* asymptote → $y = \frac{(x+5)(x+2)}{(x-5)} = x + 12 + \ldots$

So 1 vertical, 1 slant.

4. A:

I. graph of circle centered at the origin with equation $x^2 + y^2 = 16$ → symmetric about the y-axis→even function

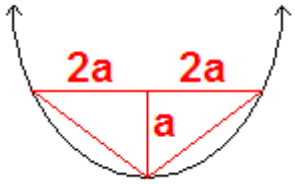
II. I $f(-x) = -f(x)$ → symmetric about the origin→ odd function

III. $f(-x) \neq f(x)$ →axis of symmetry at x=4→neither even nor odd

IV. $f(-x) \neq f(x)$ →not an even function

Only I is even.

5. C:



Triangle formed has base length 4a and height a, where a = focal length.

Area= $\frac{(4a)(a)}{2} = 2a^2$

In equation $4x^2 - 20x + 25$, $4 = \frac{1}{4a} \Rightarrow a = \frac{1}{16} \rightarrow 2a^2 = \frac{1}{128}$

6. D:

$f(x) = g(x) \rightarrow x^2 - 4x + 3 = x - 1 \rightarrow x = 1, 4$

$x = 1 \rightarrow f(1) = g(1) = 0$

$x = 4 \rightarrow f(4) = g(4) = 3$

$1 + 0 + 4 + 3 = 8$

7. E. Undefined:

$f(-1) = 3$

$g^{-1}(x) = \frac{1}{\sqrt{x-3}}$

$g^{-1}(3) = undefined$

8. B. I, IV:

Descartes' rule of signs:

# of sign changes for positive x = 4→4, 2 or 0 possible positive real roots

# of sign changes for negative x = 2→2 or 0 possible negative real roots

8 roots total→possible combination could be 4 positive, 0 negative, 4 non-real roots or 0 positive, 0 negative, 8 non-real.

9. A:
Vertex of g(x) at (-4, 1/4). Focal length = 1/4, with focus directly below the vertex since the parabola opens down. So focus at (-4,0) and vertex of f(x) at (-4,0).
For f(x), x coordinate of vertex = -4. Plugging into the equation: $(-4)^2 + 8(-4) + C = 0$
C=16

10. D:
Complex roots of polynomials must come with their conjugates. So other root of the polynomial must be 2-3i.
$(x-2-3i)(x-2+3i) = x^2 - 4x + 13$
$A(B+C) = 1(-4+13) = 9$

11. A:
$f(2) = (2)^2 + 5(2) + 2 = 16$
$g(2) = 1$
$f(2) - g(2) = 15$

12. E. 0
x+1=0 → x = -1
$2007(-1+1-1+1+\text{K}\ -1+1) = 0$

13. B:
Nelia's equation: $(x+3)(x-6) = x^2 - 3x - 18$
Yan's equation: $(x+1)(x-10) = x^2 - 9x - 10$
Correct equation: $x^2 - 3x - 10 = 0 \rightarrow x = -2, 5$

14. D:

$$f(u,v)=\frac{(u^3-v^3)(u+v)^3}{(u^2+uv+v^2)(u^4-v^4)}=\frac{(u^2-v^2)(u^2+uv+v^2)(u+v)^2}{(u^2+uv+v^2)(u^2-v^2)(u^2+v^2)}=\frac{(u^2+2uv+v^2)}{u^2+v^2}$$

$$f(u,v)=\frac{(u^2+2uv+v^2)}{u^2+v^2}\ -\frac{2uv}{u^2+v^2}=1$$

15. B:

By the rational root theorem: $\frac{\pm 3}{\pm 15},\frac{\pm 1}{\pm 15},\frac{\pm 3}{\pm 1},\frac{\pm 1}{\pm 1},\frac{\pm 3}{\pm 3},\frac{\pm 1}{\pm 3},\frac{\pm 3}{\pm 5},\frac{\pm 1}{\pm 5}$ are possible roots of the polynomial. 1/5 is possible.

16. D:

$x^3-7x+6=(x-1)(x+3)(x-2)>0$

$-3<x<1 \text{ or } x>2$

17. C:

2	1	-1	3	5
		2	2	10
	1	1	5	15

$f(x)=x^3-x^2+3x+5 \rightarrow f(1)=8$

18. D:

$(x+1)(x-2)(x-3)>0 \rightarrow -1\le x\le 2 \text{ or } 3\le x<\infty$

But denominator restrictions: $1-x^2=0\rightarrow x=\pm 1,\ x\neq\pm 1$

So domain is $(-1,1)\cup(1,2]\cup[3,\infty)$

19. A:
Sum of the reciprocals= -(-450)/27 = 50/3

20. D:
$3^{2x} - 12 \bullet 3^{x} + 27 = 0$

$(3^{x} - 9)(3^{x} - 3) = 0$

$x = 1, 2$

Product = 2

21. B:
$f(x) = x^4 - 10x^2 + 9 = (x^2 - 5)^2 - 25 + 9 = (x^2 - 5)^2 - 16$

Minimum value = -16

22. B:
$(x-1)(x+5)(2x+1) \rightarrow 1-1+1+5+2+1=9$

23. B:
$(-5)^2 - 4 \cdot 4 = 9$

24. C:
$f^{-1}(x) = \log_4(x+10)$

x+10>0 → x >-10

25. C:

$$\frac{(-x^3+6x^2-11x+6)}{1-x^2}=\frac{(x-3)(x-2)(1-x)}{(1+x)(1-x)}=\frac{(x-3)(x-2)}{(1+x)}$$

point of discontinuity at x=1→ $\frac{(1-3)(1-2)}{(1+1)}=1$

26. A:

$3-x=(-x+2)+1$

$$f(-x+2)=\frac{(-x+2)^2+9(-x+2)+12}{2(-x+2)+7}=\frac{x^2-13x+34}{-2x+11}$$

27. C:

Sum of the roots = 9→ $x^2-9x+c=0$

$$x_1=\frac{9+\sqrt{81-4c}}{2}=x_2=9-\sqrt{81-4c}$$

→ c = 18

28. D:

P:{(3,3), (4,1), (3,2), (4,4)}→same x contains more than 1 y value → not a function

29. A:

$u(-2)=0$

$$w^{-1}(x)=\frac{1}{x}+1$$

$w^{-1}(0)=undefined$

30. B:

Sum of coefficients = $(4-6)^3=-8$