

Polynomials & Rational Functions - Theta
2007 Mu Alpha Theta National Convention

For all questions, answer E. "NOTA" means none of the above answers is correct.

1. If $f(x) = x^3 + x^2 - x + 1$, find $f(-1)$.

- A. 0
C. 2

- B. 1
D. 4

E. NOTA

2. Find the equation of the line passing through the points (2,3) and (-6,-1).

A. $y = x + 1$

B. $y = \frac{x}{2} + 2$

C. $y = -x + 5$

D. $y = -\frac{x}{2} + 4$

E. NOTA

3. Characterize the asymptotes in the graph of $f(x) = \frac{x^3 + 6x^2 + 3x - 10}{x^2 - 6x + 5}$

- A. 2 vertical, 2 horizontal
C. 1 vertical, 2 horizontal

- B. 2 vertical, 1 horizontal
D. 1 vertical, 1 horizontal

E. NOTA

4. Which of the following are even functions of x?

I. $f(x) = \sqrt{16 - x^2}$

II. $f(x) = x|x|$

III. $f(x) = x^2 - 8x + 16$

IV. $f(x) = x^4 - 5x^2 + 2x + 3$

- A. I only
C. III only

- B. I, II only
D. III, IV only

E. NOTA

5. Let $f(x) = 4x^2 - 20x + 25$. Let ABC equal the triangle formed by connecting the endpoints of the latus rectum and the vertex of $f(x)$. Find the area of ABC.

- A. 1/512
C. 1/128

- B. 1/256
D. 1/64

E. NOTA

6. The graphs of the functions $f(x) = x^2 - 4x + 3$ and $g(x) = x - 1$ intersect at points (a, b) and (c, d) . Find the value of $a + b + c + d$.

- A. 5
C. 7

- B. 6
D. 8

E. NOTA

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7. $f(x) = 2x + 5$. $g(x) = \frac{1}{x^2} + 3$, for $x > 0$. Find $(g^{-1} \circ f)(-1)$.

A. 1

C. 9

B. 4

D. 16

E. NOTA

8. If $f(x) = 8x^8 - Ax^7 + Bx^5 - Cx^4 - Dx^3 + 1$, where A, B, C, D are positive values, then $f(x)$ could have:

I. 4 positive, 4 non-real roots

II. 2 positive, 4 negative, 2 non-real roots

III. 4 negative, 4 non-real roots

IV. 8 non-real roots

A. I, II only

C. II, III only

B. I, IV only

D. II, IV only

E. NOTA

9. If $f(x) = x^2 + 8x + C$ and $g(x) = -x^2 - 8x - 63/4$, find C such that the vertex of $f(x)$ lies on the focus of $g(x)$.

A. 16

C. 0

B. $\frac{63}{4}$

D. $-\frac{63}{4}$

E. NOTA

10. Given that one of the roots of the equation $Ax^2 + Bx + C = 0$ is $2+3i$, and A, B, C are relatively prime (where A is a positive number), find $A(B+C)$.

A. -17

C. 5

B. -4

D. 9

E. NOTA

11. If $f(x) = x^2 + 5x + 2$ and $g(x) = 2x - 3$, find $(f - g)(2)$.

A. 15

C. -3

B. 12

D. -5

E. NOTA

12. $f(x) = 2007x^{2007} + 2007x^{2006} + 2007x^{2005} + 2007x^{2004} + \dots + 2007x^2 + 2007x^1 + 2007x^0$. Find the remainder when $f(x)$ is divided by $x+1$.

A. 2007

C. -1

B. 1

D. -2007

E. NOTA

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13. At a Mu Alpha Theta practice session, Nelia and Yan are trying to solve a quadratic equation. Nelia writes down the equation but makes a mistake when copying down the constant term and gets that this equation's roots are $-3, 6$. Yan also tries to write down the equation but makes a mistake when copying down the x term and obtains the roots $-1, 10$. Assuming that both students solved their respective equations correctly, what were the original equation's roots?

- | | | |
|-------------|------------|---------|
| A. $-1, -3$ | B. $-2, 5$ | |
| C. $0, 9$ | D. $6, 10$ | E. NOTA |

14. $f(u, v) = \frac{(u^3 - v^3)(u + v)^3}{(u^2 + uv + v^2)(u^4 - v^4)} - \frac{2uv}{u^2 + v^2}$. Given that $|u| \neq |v|$, and $uv \neq 0$, simplify $f(u, v)$.

- | | | |
|--|--|---------|
| A. $f(u, v) = \frac{(u^3 - v^3) - 2uv}{(u^2 + v^2)}$ | B. $f(u, v) = \frac{(u + v) - 2uv}{(u^2 + v^2)}$ | |
| C. $f(u, v) = \frac{-2uv}{(u^2 + v^2)}$ | D. $f(u, v) = 1$ | E. NOTA |

15. Which of the following is a possible rational root of $f(x) = 15x^4 - ax^3 + bx^2 + cx + 3$?

Note: a, b, c are positive integers.

- | | | |
|------------------|------------------|---------|
| A. 0 | B. $\frac{1}{5}$ | |
| C. $\frac{5}{3}$ | D. 5 | E. NOTA |

16. Solve for x : $x^3 - 7x + 6 > 0$.

- | | | |
|---------------------------------|--------------------------------|---------|
| A. $(-\infty, -3) \cup (-1, 2)$ | B. $(-\infty, -3) \cup (1, 2)$ | |
| C. $(-3, -1) \cup (2, \infty)$ | D. $(-3, 1) \cup (2, \infty)$ | E. NOTA |

17.

$$\begin{array}{r} 2 \mid ? & ? & ? & ? \\ \hline 1 & 1 & 5 & 15 \end{array}$$

If the above is the result of synthetic division of a 3rd degree polynomial $f(x)$, what is $f(1)$?

- | | | |
|--------------------|-------------------|---------|
| A. $-\frac{15}{2}$ | B. $\frac{15}{2}$ | |
| C. 8 | D. 10 | E. NOTA |

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18. The domain of $f(x) = \frac{\sqrt{(x+1)(x-2)(x-3)}}{1-x^2}$ is:

- A. $(-\infty, -1) \cup (1, 3]$
B. $(-\infty, -1) \cup (1, 2] \cup [3, \infty)$
C. $(-1, 2] \cup [3, \infty)$
D. $(-1, 1) \cup (1, 2] \cup [3, \infty)$
E. NOTA

19. The sum of the reciprocals of the solutions of the equation $x^5 - 45x^4 - 8x^2 - 450x + 27 = 0$ is:

- A. $\frac{50}{3}$
B. 15
C. $\frac{5}{3}$
D. $-\frac{5}{3}$
E. NOTA

20. What is the product of the solutions of the equation $9^x - 4(3^{(x+1)}) + 27 = 0$?

- A. 27
B. 12
C. 9
D. 2
E. NOTA

21. What is the minimum value of $f(x) = (x+1)(x+3)(x-1)(x-3)$?

- A. -34
B. -16
C. 16
D. 34
E. NOTA

22. If $2x^3 + 9x^2 - 6x - 5$ is factored into the form $(ax+b)(cx+d)(ex+f)$, where a, b, c, d, e, f are integers, and a, c, e are positive. Find $a+b+c+d+e+f$.

- A. 12
B. 9
C. 7
D. -2
E. NOTA

23. Find the value of the discriminant of $x^2 - 5x + 4 = 0$

- A. 12
B. 9
C. 3
D. -3
E. NOTA

24. If $f(x) = 4^x - 10$, what is the domain of $f^{-1}(x)$?

- A. Real Numbers
B. $x > -16$
C. $x > -10$
D. $x > 0$
E. NOTA

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25. $f(x) = \frac{-x^3 + 6x^2 - 11x + 6}{1-x^2}$. The graph of $f(x)$ has a vertical asymptote and another removable discontinuity (or “hole”) at $x=c$. What is the y-value that the graph approaches near $x=c$?

A. -6
C. 1

B. -5
D. 7

E. NOTA

26. If $f(x+1) = \frac{x^2 + 9x + 12}{2x + 7}$, what is $f(3-x)$?

A. $\frac{x^2 - 13x + 34}{-2x + 11}$
C. $\frac{x^2 + 17x + 64}{2x + 15}$

B. $\frac{x^2 + x - 8}{2x - 1}$
D. $\frac{x^2 + 4x + 28}{2x + 12}$

E. NOTA

27. The sum of the roots of the equation $x^2 + bx + c = 0$ is 9. One of the roots is also twice the other. What is the value of c ?

A. -20
C. 18

B. 14
D. 20

E. NOTA

28. Let P^{-1} be a relation such that P^{-1} is the following: $\{(3,3), (1,4), (2,3), (4,4)\}$. Is P a function? If so, which is the correct set of ordered pairs for P ?

- A) Yes, P is a function. $(3,3), (4,1), (3,2), (4,4)$.
B) Yes, P is a function. $(3,3), (1,4), (2, 3), (4, 4)$.
C) Yes, P is a function. $(3,3), (4,1), (3,2), (4,4)$.
D) No, P is not a function.
E) NOTA

29. $u(x) = x^2 - 4$, $v(x) = x + 3$, $w(x) = \frac{1}{x-1}$. Find $v(w^{-1}(u(-2)))$.

A. undefined
C. $\frac{31}{8}$

B. 3
D. 4

E. NOTA

30. The sum of the coefficients in the binomial expansion of $(4x^3 - 6y^3)^3$ is:

A. -512
C. 8

B. -8
D. 512

E. NOTA