Theta Sequences and Series
2007 Mu Alpha Theta National Convention

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For all questions, NOTA means "none of the above" answers is correct.

1) Find the common difference of the arithmetic sequence: $-5, -14, -23, \ldots$
A) –9 B) –5 C) 5 D) 9 E) NOTA

2) Find the common ratio of the geometric sequence of real numbers: $-\frac{25}{12}, -\frac{5}{4}, -\frac{3}{4}, \ldots$
A) $-\frac{3}{5}$ B) $-\frac{2}{5}$ C) $\frac{2}{5}$ D) $\frac{3}{5}$ E) NOTA

3) Find the sum of the first nine terms of an arithmetic sequence of real numbers with 2 as the fifth term and 8 as the thirteenth term.
A) 5 B) 18 C) 24 D) 45 E) NOTA

4) The sum of the first six terms of a geometric sequence with 2 as the third term and 8 as the seventh term can be expressed as $a\sqrt{b}+c$ in simplest form. Find $a+b+c$.
A) 12 B) 16 C) 24 D) 63 E) NOTA

5) Given a sequence of real numbers: 2, __, 6
Let X_1 be the missing value that makes the sequence an arithmetic sequence and X_2 be the missing value that makes the sequence a geometric sequence. Find $\frac{X_1}{X_2}$.
A) $\frac{\sqrt{3}}{2}$ B) 1 C) $\frac{2\sqrt{3}}{3}$ D) $\sqrt{3}$ E) NOTA

6) Alex decides to write down a number containing only ones and zeros. The first part of the number looks like the following: 101001000100001000001000000100000001…..
She loves math so much that she decides to write the number so it has length sufficient to have twenty-five digits equal to 1 with 1 as the first digit and 1 as the last digit (101 has two digits equal to 1; 101001 has three digits equal to 1; etc.). How many digits long is this number?
A) 325 B) 350 C) 625 D) 650 E) NOTA

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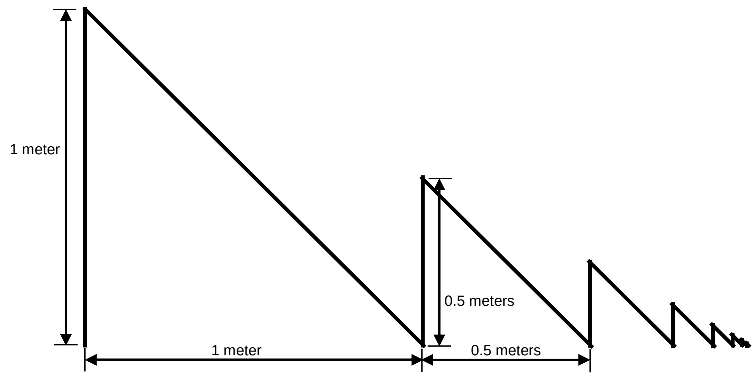
7) An ant decides to walk in a very interesting pattern. It first walks 1 meter north, stops, and then walks southeast until it is exactly 1 meter east of its starting location. It then walks 0.5 meters north, stops, and then walks southeast again until it is 0.5 meters east of the second starting location. The distances are halved again and the pattern repeats. If the ant continued this pattern for an infinite amount of time, what is the total distance in meters it would travel if it were to follow this pattern?



A) $1+\sqrt{2}$ B) $2+\frac{\sqrt{2}}{2}$ C) $2+\sqrt{2}$ D) $2+2\sqrt{2}$ E) NOTA

8) The sum of the first twelve terms of an arithmetic sequence of real numbers is 60. The first term is –6. What is the sum of the first twenty terms?

A) 108 B) 160 C) $\frac{740}{3}$ D) 260 E) NOTA

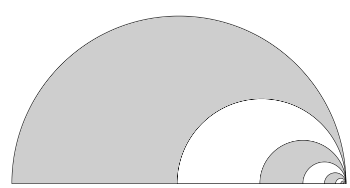
9) A 30-car train has a certain number of passengers. Each car has 24 more passengers than the car in front of it. Also, on each car there are twice as many women as there are men. If the first car has 10 men, then how many more women are there than men on the entire train?

A) 3780 B) 3900 C) 5670 D) 5850 E) NOTA

10) The n^{th} term of a given sequence is $a_n = n^n$. What is the units digit of $\sum_{n=1}^{10} a_n$?

A) 3 B) 4 C) 7 D) 8 E) NOTA

11) The figure shows a pattern of semicircles inscribed in one another until they are infinitely small. The radius of any semicircle is half the radius of the next largest. If the radius of the largest semicircle is R, find the area of the shaded region in terms of R.



A) $\frac{\pi}{4}R^2$ B) $\frac{\pi}{3}R^2$ C) $\frac{2\pi}{5}R^2$ D) $\frac{4\pi}{9}R^2$ E) NOTA

12) The n^{th} term of a given sequence is $a_n = \frac{n-1}{n+1}$. Find: $a_2 \cdot a_3 \cdot \ldots \cdot a_{100}$.

A) $\frac{1}{10100}$ B) $\frac{1}{9900}$ C) $\frac{1}{5050}$ D) $\frac{1}{4950}$ E) NOTA

13) Colin has the following schedule on the weekends. He wakes up at 8:00am and cycles through three activities the entire day. Once waking up, he first does homework for 5 minutes, then works on his computer for 6 minutes, and then eats some food for 7 minutes. He then repeats this schedule, except that he doubles the time for each activity (i.e. the next cycle would be 10 minutes for homework, 12 minutes on the computer, and 14 minutes eating food). The cycle continues non-stop until 10:00pm, upon which he decides to go to sleep. How many complete cycles does Colin complete in one day? (A complete cycle is homework-computer-eating) Assume that there is no transition time between activities.

A) 6 B) 7 C) 8 D) 9 E) NOTA

14) Evaluate the geometric series: $1 + \frac{i+1}{2} + \frac{i}{2} + \frac{-1+i}{4} - \frac{1}{4} \pm \ldots;\ i = \sqrt{-1}$

A) $1-i$ B) $\frac{1-i}{2}$ C) $\frac{1+i}{2}$ D) $1+i$ E) NOTA

15) Let $\{a_1, a_2, a_3, \ldots, a_n, \ldots\}$ and $\{b_1, b_2, b_3, \ldots, b_n, \ldots\}$ be two different convergent geometric sequences. Let $c_n = a_n + b_n$ ($c_1 = a_1 + b_1, c_2 = a_2 + b_2$, etc.) Which of the following MUST be true? (NOTE: convergent means that the sequence approaches a finite value as n becomes large).

I) $\{c_1, c_2, c_3, \ldots, c_n, \ldots\}$ is a geometric sequence
II) $\{c_1, c_2, c_3, \ldots, c_n, \ldots\}$ is an arithmetic sequence
III) $\{c_1, c_2, c_3, \ldots, c_n, \ldots\}$ is a convergent sequence

A) I only B) II only C) III only D) I & III E) NOTA

16) A quadratic sequence with real coefficients, $a_n = An^2 + Bn + C$, satisfies the following: $a_1 = 4$, $a_2 = 7$, and $a_3 = 3$. Find $|A| + |B| + |C|$.

A) 4 B) 12 C) 19 D) 23 E) NOTA

17) David decided to make a sequence that follows the given pattern: $a_n = a_{n-1} - a_{n-2}$; $a_1 = -2$, and $a_2 = 1$. Find a_7.

A) –3 B) –2 C) –1 D) 1 E) NOTA

18) Evaluate: $\sqrt{6 + \sqrt{6 + \sqrt{6 + \ldots}}}$

A) 2 B) $\sqrt{6}$ C) 3 D) 6 E) NOTA

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19) Find the sum of the smallest ten odd square numbers. (Assume that 1 is the first)
A) 385 B) 579 C) 969 D) 1330 E) NOTA

20) A recursive relation is defined as follows: $A_{n+1} = \frac{1}{3}A_n$, $A_1 = \frac{1}{2}$. Evaluate $\sum_{n=1}^{\infty} A_n$
A) 1 B) $\frac{3}{4}$ C) $\frac{5}{6}$ D) Does not exist E) NOTA

21) The n^{th} term of a given sequence is $a_n = \log_3(n+1)$. Find $\sum_{n=1}^{2007} a_n$.
A) $\log_3(2007!)$ B) $\log_3(2007!+1)$ C) $\log_3(2007!)+1$ D) $\log_3(2008!)$
E) NOTA

22) Shown is a geometric sequence of real numbers with two unknown elements. Find $a+b$.
$\frac{1}{4}, a, b, \frac{2}{27}$
A) $\frac{11}{36}$ B) $\frac{7}{18}$ C) $\frac{5}{12}$ D) $\frac{15}{16}$ E) NOTA

23) Oscar drops a ball from 12 feet off the ground. The ball's height on the each bounce is one-fifth its previous height. If the ball were to bounce forever, what would be the total vertical distance in feet traveled by the ball?
A) 15 B) 18 C) 27 D) 30 E) NOTA

24) Given an arithmetic sequence whose first term is π and whose second term is e, find the 10^{th} term of the sequence.
A) $9e-9\pi$ B) $9e-8\pi$ C) $9\pi-9e$ D) $9\pi-8e$ E) NOTA

25) Given a geometric sequence whose n^{th} term is $c_n = a \cdot b^n$, where a and b are positive integers and $0 < a < 1$ and $0 < b < 1$, how many of the following statements MUST be true?

I) $\sum_{n=0}^{\infty} c_n$ is a finite value II) $\sum_{n=0}^{\infty} \frac{1}{c_n}$ is not a finite value

III) $c_n < 1$ for any value of n IV) $\sum_{n=0}^{\infty} (c_n)^2 < \sum_{n=0}^{\infty} c_n$

A) 1 B) 2 C) 3 D) 4 E) NOTA

26) Which of the following describes the given sequence?
2, 4, 8, 14, 22, …
A) Arithmetic only B) Geometric only C) Neither arithmetic nor geometric
D) Both arithmetic and geometric E) NOTA

27) Find the sum of the infinite series: $\left(\frac{1}{2}-\frac{1}{3}\right)+\left(\frac{1}{4}-\frac{1}{9}\right)+\left(\frac{1}{8}-\frac{1}{27}\right)+\ldots$

A) $\frac{1}{4}$ B) $\frac{1}{3}$ C) $\frac{2}{5}$ D) $\frac{1}{2}$ E) NOTA

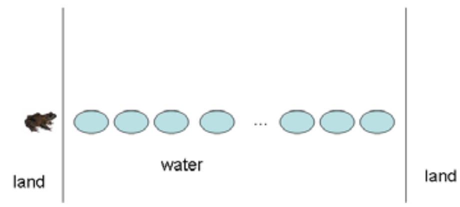
28) Find the coefficient of the x^8y^3 term of the binomial expansion $(3x^4-4y)^5$.

A) –5760 B) –576 C) 3820 D) 4320 E) NOTA

29) Evaluate: $\sum_{n=1}^{2007}\log_{10}\left(\frac{n}{n+1}\right)$

A) $-\log_{10}2008$ B) $-\log_{10}2007$ C) $\log_{10}2007$ D) $\log_{10}2008$
E) NOTA

30) Bradley the frog is jumping across a river. There are 50 lily pads spanning from one edge to the other as shown in the figure. Bradley wants to cross the river by following a certain pattern. He first makes 5 hops forward, then one backwards, then 5 forward again, then one backward, and so forth until he has made it to the other side. (NOTE: a hop is equal to the distance of a jump from the center of one lily pad to an adjacent lily pad). If he continues this pattern, how many lily pads does he land on more than once? (Assume that once he's made it to the other side he doesn't jump back anymore.)



A) 18 B) 20 C) 22 D) 24 E) NOTA