

Question #1

A. $\frac{27}{4}(4) - 23\frac{1}{4} = \frac{15}{4}$

B. Draw Picture: Your picture should create two small triangles and a small square;

$$\frac{1}{2}(5)(1) + 25 + \left(25 - \frac{1}{2}(5)\left(\frac{5}{3}\right)\right) = \frac{5}{2} + 25 + 25 - \frac{25}{6} = \frac{145}{3}$$

C. Draw Picture: $4x + 2y = 10 \rightarrow y = 5 - 2x \rightarrow 2x + x^2 - k - 5 = 0 \rightarrow x^2 + 2x - k - 5$
 $(x-1)(x+3) \rightarrow -3 = -k - 5 \rightarrow k = -2 \rightarrow (x-2)(x+4) \rightarrow -8 = -k - 5 \rightarrow k = 3$

D. Rate times time equals distance: $60\left(\frac{d}{200} + \frac{4d}{5r}\right) = d \rightarrow 60\left(\frac{1}{200} + \frac{4}{5r}\right) = 1$

$$\frac{r+160}{200r} = \frac{1}{60} \rightarrow 60r + 9600 = 200r \rightarrow 140r = 9600 \rightarrow r = \frac{480}{17}$$

Question #2

A. $2^{37} - 2^1 = \log x + \log x^2 + \log x^4 + \dots + \log x^{2^{35}} \rightarrow 2(2^{36} - 1) = \log x^{(1+2+4+\dots+2^{35})}$
 $= \log x^{2^{35}} \rightarrow \log x = 2 \rightarrow x = 100$

B. $3^{3x} - 3^{3y} = 480 \rightarrow (3^x - 3^y)(3^{2x} + 3^{x+y} + 3^{2y}) = 480$
 $10(12 + 3^{x+y}) = 480 \rightarrow 12 + 3^{x+y} = 48 \rightarrow 3^{x+y} = 36$

C. $\frac{A}{k+2} + \frac{B}{k-2} \rightarrow A(k-2) + B(k+2) = 8 \rightarrow A+B=0 \rightarrow -2A+2B=8$

$$A = -2 \rightarrow B = 2 \rightarrow \frac{2}{k-2} - \frac{2}{k+2} = \left(\frac{2}{1} - \frac{2}{5}\right) + \left(\frac{2}{2} - \frac{2}{6}\right) + \left(\frac{2}{3} - \frac{2}{7}\right) + \dots$$

$$2 + 1 + \frac{2}{3} + \frac{1}{2} = \frac{25}{6}$$

D. $(\sqrt{a} - \sqrt{b})(\sqrt{a} + \sqrt{b}) = 12 \rightarrow (\sqrt{a} - \sqrt{b})(8) = 12 \rightarrow (\sqrt{a} - \sqrt{b}) = \frac{3}{2}$

$$2\sqrt{a} = \frac{19}{2} \rightarrow \sqrt{a} = \frac{19}{4} \rightarrow a = \frac{361}{16}$$

Question #3

A. Draw Picture: $\frac{x}{14} = \frac{18}{19+x} \rightarrow 19x + x^2 = 252 \rightarrow (x+28)(x-9) = 0 \rightarrow x = 9$

B. Draw picture: $\frac{2}{10} = \frac{3}{3+x} \rightarrow 6 + 2x = 30 \rightarrow 2x = 24 \rightarrow x = 12$

C. Draw Picture: $12 - x + 16 - y + x + y = 28$

D. Draw Picture: $5(12) = 4(x+7) \rightarrow 60 = 4x + 28 \rightarrow 4x = 32 \rightarrow x = 8 \rightarrow 3(8) = 6y \rightarrow y = 4$

Question #4

A. The ratio of the distances must be constant:

$$\frac{700}{x+300} = \frac{x}{1100+x} \rightarrow x^2 + 300x = 770000 + 700x \rightarrow x^2 - 400x - 770000 = 0$$

$$(x-1100)(x+400) = 0 \rightarrow x = 1100 \rightarrow 100 + 700 = 1800$$

B.

$$2x - 3y = 6 \rightarrow m = \frac{-3}{2} \rightarrow 3x + 2y = 28 \rightarrow \frac{|2 \cdot 4 + (-3) \cdot 8 - 6|}{\sqrt{2^2 + 3^2}} = \frac{|2x - 3y - 6|}{\sqrt{2^2 + 3^2}} \rightarrow 22 = 2x - 3y - 6 \rightarrow 3x + 2y = 28$$

Solve the system of equations and you get $x = \frac{140}{13}$

C. $10 - |3x^2 - 9| > 0 \rightarrow -10 < 3x^2 - 9 < 10 \rightarrow -1 < 3x^2 < 19 \rightarrow x^2 < \frac{19}{3} \rightarrow \left(\frac{-\sqrt{57}}{3}, \frac{\sqrt{57}}{3} \right)$

D. Draw picture: $4p(x-h) = (y-k)^2 \rightarrow -16(x-6) = (y+3)^2 \rightarrow -16x + 96 = 81 \rightarrow -16x = -15 \rightarrow x = \frac{15}{16}$

Question #5

A. Factor: $(3x - 2y + 2)(2x - 5y + 1) \rightarrow n = -2$

B. Draw Picture: $VXAB : VXZY$ Medians intersect two thirds of the way from vertex:

$$\frac{XW}{XE} = \frac{2}{3} \rightarrow \left(\frac{2}{3} \right)^2 = \frac{4}{9}$$

C. Draw Picture: $VAYZ : VACB \rightarrow \text{Area of } VAYZ = \frac{1}{2} VACB \rightarrow \left(\frac{x}{12} \right)^2 = \frac{1}{2} \rightarrow x^2 = 72 \rightarrow x = 6\sqrt{2}$

D. Rate times Time equals Work:

$$\frac{6}{x+3} + \frac{6}{x} = \frac{7}{x-3} \rightarrow 6x(x-3) + 6(x^2-9) = 7x(x+3) \rightarrow 5x^2 - 39x - 54 = 0 \rightarrow$$

$$(5x+6)(x-9) = 0 \rightarrow x = 9$$

Question #6

$$A. \frac{x+1}{x-1} \left[x^2 - 1 + \frac{x-1}{x+1} \right] = (x+1)^2 + 1 \rightarrow x^2 + 2x + 2$$

$$B. \frac{(x+2)^3(x^2+4x+3)(x^2-6x+8)(4)}{(x^2+x-2-x^2+x+6)(x^2+x-2+x^2-x-6)} \rightarrow \frac{(x+2)^3(x+1)(x+3)(x-4)(x-2)4}{2(x+2)2(x-2)(x+2)} \rightarrow x-4$$

$$C. \frac{(a^2-9-c^2-6c)(a^2-9-c^2+6c)}{(a^2-6a+9-c^2)(a^2-c^2+3a+3c)} \rightarrow \frac{[a^2-(c^2+6c+9)][a^2-(c^2-6c+9)]}{[(a-3)^2-c^2][(a-c)(a+c)+3(a+c)]}$$

$$\frac{(a-c-3)(a+c+3)(a-c+3)(a+c-3)}{(a-3-c)(a-3+c)(a+c)(a-c+3)} \rightarrow \frac{a+c+3}{a+c}$$

$$D. \frac{[x^2-(4y^2-4y+1)](x+2y)}{(x^2-4y^2-x+2y)(x^2-4y^2+x+2y)} = \frac{x^2-(2y-1)^2}{(x-2y)(x+2y)-(x-2y)}$$

$$\frac{(x-2y+1)(x+2y-1)(x+2y)}{(x-2y)(x+2y-1)(x+2y)(x-2y+1)} = \frac{1}{x-2y}$$

Question #7

$$A. \text{Rate times time equals distance: } 1 + \frac{b-c+2}{b+c} = \frac{2}{c} \rightarrow (b+c)c + c(b-c+2) = 2(b+c) \rightarrow$$

$$bc + c^2 + bc - c^2 + 2c = 2b + 2c \rightarrow 2bc - 2b = 0 \rightarrow b(c-1) = 0 \rightarrow c = 1$$

$$B. \text{Solve a System of Equations: } x + y + 2z = 78 \quad x + 2y + 4z = 140 \quad 40x + 60y + 100z = 3880$$

$$z = 24$$

$$C. \frac{45+4-36-12}{18} = \frac{1}{18} : \text{so 18hrs equals one person: } 18(10) = 180$$

$$D. \text{Rate times time equals distance: } ry = 3600 \rightarrow (r+60)(y-3) = 3600 \rightarrow \left(\frac{3600}{y} + 60 \right) (y-3) = 3600$$

$$3600 - \frac{10800}{y} + 60y - 180 = 3600 \rightarrow 60y^2 - 180y - 10800 = 0$$

$$y^2 - 3y - 180 = 0 \rightarrow (y-15)(y+12) = 0 \rightarrow y = 15$$

Question #8

A. Two show the same number or all three show the same number:

$$6 + C_2^6 \cdot C_1^6 = \frac{6+90}{216} = \frac{4}{9}$$

$$B. C_2^3 \cdot C_2^2 + C_2^3 \cdot C_2^3 / C_2^6 \cdot C_2^5 \rightarrow \frac{2}{25}$$

$$C. \frac{2C_4^8}{C_5^{10}} = \frac{5}{9}$$

D. Since two of the five heads are on the same coin, two will be your numerator. The condition of heads narrows your denominator down to five so the answer is $\frac{2}{5}$

Question #9

A. First find the equation of the perpendicular line: $3x + 4y = 0$ then find the intersection of this line and the given line by solving a system of equations: $\left(\frac{-4}{5}, \frac{3}{5}\right)$. Then find the midpoint of this point and the given

point: $\left(\frac{8}{5}, \frac{-6}{5}\right)$

$$B. (-3)(84)(-8) + 5(36)(4) + 16(126) = 4752$$

$$C. A = \frac{6+B}{2} \rightarrow \sqrt{16A} = B \rightarrow 2A - 6 = B \rightarrow \sqrt{16A} = 2A - 6$$

$$16A = 4A^2 - 24A + 36 \rightarrow A^2 - 10A + 9 = 0 \rightarrow (A-9)(A-1) = 0 \rightarrow 9+1=10$$

$$D. y^2 + \sqrt{5} = \sqrt{5} + \sqrt{6} \rightarrow y^2 = \sqrt{6} \rightarrow y^4 = 6$$

Question #10

A. Use your properties of determinants: First add row one to row three, then expand by the third column since it now has three zeros. This will give you -2 multiplied by its minor which gives you an answer of -66.

B. Use your properties of determinants: Multiply third row by -2 and add it to the fourth row. Then expand by the fourth column because it will now have three zeros. You will get -1 multiplied by its minor which will give an answer of -27

C. Sum of digits must be divisible by three: 111, 444, 555, 666, 888 = 5

114, 441, 558, 885, = 4 times three which equals 12: 156, 168, 456, 468 equals four times six which is 24. Add 5, 12, and 24 to get 41.

D. $8! \left(\frac{3}{4} \right) \left(\frac{1}{2! \bullet 2!} \right) = 7560$: Can't lead with zero: Zeros and threes must be divided out because won't create different number.