Theta State Bowl Question #1 2007 MAΘ National Convention

A. Suppose that exactly three answers to a standard Mu Alpha Theta test are E. The remaining 27 answer choices are randomly chosen to be A, B, C, or D. What is the expected score if a student were to bubble B to all 30 problems? (Expected score need not be an integer.)

B. A square of side length 10 units is in the xy-plane with the intersection of the diagonals of the square at the origin. The line x=y contains one of the square's diagonals. The solution to the system $\left\{(x, y): x-3y \geq 0 \; and \; x+2y \leq 5\right\}$ is shaded on that same plane. Find the area of the shaded region that is contained within the square region.

C. A parabola defined by $y = x^2 - k$ for k>0 contains two of the vertices of rectangle FAST. One side of FAST is on the x-axis. Rectangle FAST has perimeter 10 units and its sides are of positive integral lengths. Give the largest possible real value of k.

D. Because of traffic, the Buchholz school bus only averaged 40 mph for the first fifth of the trip to nationals, but they averaged 60 mph for the whole trip. What was their average speed in mph for the last 80% of their trip?

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Theta State Bowl **Question #2** **2007 MAΘ National Convention**

A. Solve for all positive value(s) of x: $2^{37} = 2 + \sum_{n=0}^{35} \log\left(x^{2^n}\right)$

B. Given $27^x - 9^{\frac{3y}{2}} = 480$, $3^x - 3^y = 10$, and $9^x + 27^{\frac{2y}{3}} = 12$, find the value of 3^{x+y}.

C. Express the following sum as a quotient of two integers in simplest form $\sum_{k=3}^{\infty} \frac{8}{(k+2)(k-2)}$.

D. If $\sqrt{a} + \sqrt{b} = 8$ and $a - b = 12$, then what is the value of a?

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Theta State Bowl　　　Question #3　　　2007 MAΘ National Convention

A. There exists a triangle ABC with points D and E on $\overline{BC}$ and $\overline{AB}$ respectively, such that $\triangle BED \sim \triangle BCA$. If BD = 14, DC = 4, and AE = 19, what does BE=?

B. There exists a triangle ABC with points D, E, and F on sides $\overline{AB}, \overline{AC}, and\ \overline{BC}$ respectively. If $\overline{DE} \parallel \overline{BC}$ and $\overline{EF} \parallel \overline{AB}$ and AD = 2, DB = 8, and BF = 3, then FC=?

C. In triangle ABC, points D and E lie on $\overline{AB}$ and $\overline{BC}$ respectively, such that $\overline{DE} \parallel \overline{AC}$. If point F lies on $\overline{DE}$ such that $\overline{CF}$ bisects $\angle ECA$ and $\overline{AF}$ bisects $\angle DAC$ and AC = 24, AB =16 and BC = 12. Find the perimeter of triangle BDE.

D. Given circle O with a point X outside the circle such that $\overline{XY}$ and $\overline{XZ}$ are secants of circle O. If secant $\overline{XY}$ and $\overline{XZ}$ intersect the circle at points A and B respectively and $\overline{YC}$ is a chord that intersects $\overline{XZ}$ at point D and DZ = 3, YD = 6, XA = 5, AY = 7, and XB = 4, find CD.

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Theta State Bowl **Question #4** **2007 MAΘ National Convention**

A. Two flies go back and forth across a room with constant but different speeds, turning at the opposite wall without loss of time. They leave opposite walls at the same instant, meeting for the first time 700cm from one wall and meet for the second time 300 cm from the opposite wall. What is the width of the room?

B. A line is given by the equation $y=\frac{2}{3}x-2$. A point K at coordinates $(4,8)$ is reflected over this line. What is the abscissa of the reflected point?

C. Given: $f(x)=\log_6\left(10-\left|3x^2-9\right|\right), g(x)=6^x, and\ h(x)=x+10$. $h\left(g\left(f(k)\right)\right)$ is defined for what values of k (give answer in interval notation)?

D. If W is the locus of points $Z(x,y)$ such that the distance from Z to the point P$(2,-3)$ is equal to the distance from Z to the line x=10. If the point $(N,-12)$ is in the set W then N = ?

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Theta State Bowl **Question #5** **2007 MAΘ National Convention**

A. For what value of n will the graph of the equation $6x^2 - 19xy + 10y^2 + 7x - 12y = n$ form two intersecting lines?

B. W is the intersection point of the medians of triangle XYZ. A line through W parallel to $\overline{YZ}$ divides the triangle into two regions. What is the ratio of the area of the smaller triangle to the area of the larger triangle?

C. In triangle ABC, altitude $\overline{AD}$ is 12 units long. Through point E of $\overline{AD}$ a line is drawn parallel to $\overline{BC}$, dividing the triangle into two regions with equal areas, Find AE.

D. A simpleton takes three more days to write a Mu test than a sempi. A master test writer takes three fewer days than a sempi to write a Mu test. The master test writer can do as much work in seven days as the simpleton and the sempi working together can accomplish in six days. How many days would it take the sempi alone to write a Mu test?

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Theta State Bowl **Question #6** **2007 MAΘ National Convention**

A. Simplify: $\left(\frac{x+1}{x-1}\right)\left[\frac{x^4-1}{x^2+1}+\frac{(x-1)^2}{x^2-1}\right]$

B. Simplify: $\frac{\left[(x+2)^5-(x+2)^3\right](4x^2-24x+32)}{\left[(x^2+x-2)^2-(x^2-x-6)^2\right](x^3+6x^2+11x+6)}$

C. Simplify: $\frac{(a^2-3^2-c^2)^2-4(3c)^2}{(a^2-c^2-6a+9)(a^2+3a+3c-c^2)}$

D. Simplify: $\frac{(x^2-4y^2+4y-1)(x+2y)}{(x^2-x-4y^2+2y)(x^2+2y+x-4y^2)}$

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Theta State Bowl **Question #7** **2007 MAΘ National Convention**

A. After leaving his river house, a canoeist rowing upstream passes a log 2 miles upstream from his house. The canoeist rows upstream for one more hour and then rows back to his house, arriving at the same time as the log. How fast was the current flowing in miles per hour?

B. The Wiggles and Bear are touring this summer. They are selling three types of tickets for their performance: a single adult ticket costs $40, a ticket for one adult and one child costs $60, and a ticket for two adult with two children costs $100. The Wiggles and Bear collected $3880 in ticket sales from 140 people, 78 of whom were adults. How many two adult with two children tickets did they sell?

C. In Birmingham a birth occurs on the average every 24 minutes and a death every 30 minutes. A resident moves out of the city every 1.5 hours, and a new person moves into the city every 4.5 hours. How many hours does it take on the average for the population to increase by 10 people?

D. In one year The Road Runner increased his running speed by 60 miles per minute. At the end of that year it took him 3 minutes less time to run a 3600 mile course than it took him at the beginning of the year. How many minutes did it take The Road Runner originally to run the course?

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Theta State Bowl Question #8 2007 MAΘ National Convention

A. Three dice are thrown. What is the probability that two or more of the dice show the same number?

B. Two letters are chosen at random from the word CHEESE and two are chosen at random from the word ENTER. What is the probability that the selection contains 4 E's or no E's?

C. Five letters are chosen at random from the last 10 letters of the alphabet. What is the probability that the selection contains X or Y but not both?

D. There are four quarters in a piggy bank, one of which is a two-headed coin. If you break the bank open and grab one quarter at random, what is the probability that you have selected the two-headed quarter, given that you looked at one side of the coin and you see heads?

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Theta State Bowl **Question #9** **2007 MAΘ National Convention**

A. Find the point that is halfway between the point $(4,-3)$ and the line $4x-3y=-5$.

B. What is the coefficient of x^7 in the expansion of $(x^2-3x+5)(x-2)^9$?

C. Find the sum of all of the values for A for which $6, A, B, 16$ is a sequence whose first three terms are arithmetic and whose last three are geometric.

D. One leg of a right triangle has length $\sqrt[4]{5}$ and the hypotenuse of the right triangle is equal to $\sqrt[4]{11+2\sqrt{30}}$. If the length of the other leg is y, then what is the value of y^4 ?

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Theta State Bowl Question #10 2007 MAΘ National Convention

A. Evaluate the determinate; $\begin{vmatrix} 3 & 4 & -2 & 1 \\ 5 & 1 & 0 & -1 \\ 0 & -3 & 2 & 3 \\ 4 & -1 & 0 & -5 \end{vmatrix}$

B. Evaluate the determinate: $\begin{vmatrix} 3 & -2 & 2 & 0 \\ 1 & -2 & 1 & 0 \\ 4 & 3 & 6 & 1 \\ -1 & 1 & 2 & 2 \end{vmatrix}$

C. How many multiples of 3 between 100 and 1000 can be formed from the digits 1, 4, 5, 6, and 8?

D. How many different 8 digit numbers can be formed if you are only allowed to use the numbers from the set $\{0,0,3,3,6,7,8,9\}$? A number may be selected only once.

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