The acronym "NOTA" stands for "None Of The Above". The standard form for a conic section (possibly rotated) with center (h, k) is $A(x-h)^2 + B(x-h)(y-k) + C(y-k)^2 = 1$.

1. Give the area of the curve given by $\frac{x^2}{2019^2} + \frac{y^2}{4} = 1$.

A. 8076π B. 4038π C. 8076 D. 4038 E. NOTA

2. Which nondegenerate conic is represented by $x^2 + xy + y^2 = 1$?

A. Parabola B. Noncircular ellipse C. Hyperbola D. Circle E. NOTA

3. Give the directrix of $2y = x^2 - 4y + 2$.

A. $x = \frac{11}{6}$ B. $y = \frac{11}{6}$ C. $x = -\frac{7}{6}$ D. $y = -\frac{7}{6}$ E. NOTA

4. Which of the following is a focus of $4x^2 - 9y^2 - 8x - 54y - 221 = 0$?

A. $\left(1 - 2\sqrt{13}, 3\right)$ B. $\left(2\sqrt{13} - 1, 3\right)$ C. $\left(1, -3 + \sqrt{13}\right)$ D. $\left(1, 3 + \sqrt{13}\right)$ E. NOTA

5. Which of the following is an asymptote of $4x^2 - 9y^2 - 8x - 54y - 221 = 0$?

A. $y = -\frac{2}{3}x + \frac{11}{3}$ B. $y = \frac{3}{2}x - \frac{9}{2}$ C. $y = -\frac{3}{2}x + \frac{3}{2}$ D. $y = -\frac{2}{3}x - \frac{7}{3}$ E. NOTA

6. Give the eccentricity of $4x^2 - 9y^2 - 8x - 54y - 221 = 0$.

A. $\frac{\sqrt{13}}{3}$ B. $\frac{\sqrt{13}}{2}$ C. $\frac{\sqrt{5}}{3}$ D. $\frac{\sqrt{5}}{2}$ E. NOTA

7. What is the area formed by the rectangle that has two of its sides as the latus rectums of $4x^2 - 9y^2 - 8x - 54y - 221 = 0$?

A. $\frac{32\sqrt{5}}{3}$ B. $\frac{64\sqrt{5}}{3}$ C. $\frac{32\sqrt{13}}{3}$ D. $\frac{64\sqrt{13}}{3}$ E. NOTA

8. The x and y intercepts of $-2+4x+5x^2-4y+2xy+y^2=0$ are $(x_1,0),(x_2,0),(0,y_1)$, and $(0,y_2)$ such that $x_1<x_2$ and $y_1<y_2$. What is $5x_1y_1$?

A. $-4-2\sqrt{6}-2\sqrt{14}-2\sqrt{21}$ C. $-4-2\sqrt{6}+2\sqrt{14}+2\sqrt{21}$ E. NOTA

B. $-4+2\sqrt{6}-2\sqrt{14}+2\sqrt{21}$ D. $-4+2\sqrt{6}+2\sqrt{14}-2\sqrt{21}$

9. Rotating the coordinate axes by an angle of θ will eliminate the xy term in the following equation: $-2+4x+5x^2-4y+2xy+y^2=0$. What is $\tan(2\theta)$?

A. $\frac{1}{2}$ B. $\frac{1}{3}$ C. 2 D. 3 E. NOTA

10. Find the center of $-2+4x+5x^2-4y+2xy+y^2=0$.

A. $(-2,2)$ B. $(-1,3)$ C. $\left(-\frac{2}{5},2\right)$ D. $\left(\frac{2}{5},-2\right)$ E. NOTA

11. Determine the area of the ellipse given by $-2+4x+5x^2-4y+2xy+y^2=0$.

A. 5π B. 2π C. π D. $\frac{\pi}{4}$ E. NOTA

12. How many petals are there on the polar plot $r=\frac{1}{2019}\left(ie^{5i\theta}-ie^{-5i\theta}\right)\left(e^{-5i\theta}+e^{5i\theta}\right)$?

A. 0 B. 5 C. 10 D. 20 E. NOTA

13. Which of the following is a polar representation of $x^2-2xy+y^2-2x+2y+1=0$?

A. $\frac{\sqrt{2}}{2\sin\left(\frac{\pi}{4}-\theta\right)}$ B. $\frac{\sqrt{2}}{2\sin\left(\frac{\pi}{4}+\theta\right)}$ C. $\frac{\sqrt{2}}{\cos\left(\frac{\pi}{4}-\theta\right)}$ D. $\frac{\sqrt{2}}{\cos\left(\frac{\pi}{4}+\theta\right)}$ E. NOTA

14. Give the eccentricity of $2019!\,(x-2018)^2+2019!\,(y+2020)^2=2019x+2019y+2\cdot 2019!\,xy+e$

A. 0 B. 1 C. 2019! D. $\frac{1}{2019!}$ E. NOTA

15. What type of conic is represented by the polar equation $r=\frac{\pi^\pi}{e^2-e^2\cos(\theta)}$?

A. Parabola B. Hyperbola C. Ellipse D. Line E. NOTA

16. Identify the polar graph $r=3+2\cos\left(\theta+\frac{\pi}{4}\right)$.

A. Convex Limacon B. Dimpled Limacon C. Cardioid D. Looped Limacon E. NOTA

17. For all real values of a, the parabola $y=2x^2+ax+3a$ passes through the common point (p,q). Find $p+q$.

A. 6 B. 12 C. 15 D. 21 E. NOTA

18. Consider the point $P(-2,1)$ on the ellipse given by the equation $4x^2+y^2+8x-2y+1=0$. There are two points (a,b) and (a,c) on the ellipse whose distance from P is maximal. What is a?

A. $-\frac{1}{3}$ B. $-\frac{3}{4}$ C. $-\frac{1}{2}$ D. $-\frac{2}{3}$ E. NOTA

19. There are four lines that are common tangents to the circles $(x-2019)^2+(y+2019)^2=1$ and $(x-2025)^2+(y+2019)^2=4$. What is the sum of the slopes of these four lines?

A. 2019 B. $\frac{1}{2019}$ C. -2019 D. 0 E. NOTA

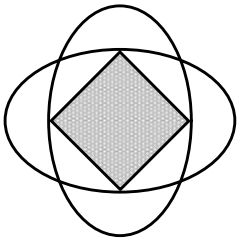
20. Compute the distance between $2x+3y=1$ and $-4x-6y=12$.

A. $\frac{7}{\sqrt{13}}$ B. $\frac{7}{\sqrt{5}}$ C. $\frac{7}{\sqrt{2}}$ D. 7 E. NOTA

21. Compute the distance between the 3D lines m and n parameterized by t such that $m(t) = \langle 2, -1, 3\rangle t + \langle -1, 1, 1\rangle$ and $n(t) = \langle 3, 2, 1\rangle t + \langle 2, 0, -1\rangle$.

A. 0 B. $2\sqrt{3}$ C. $\frac{5\sqrt{3}}{3}$ D. $7\sqrt{3}$ E. NOTA

22. The figure to the right shows two ellipses whose major axes are perpendicular to each other. Each ellipse passes through the other ellipse's foci. All four foci form the vertices of a square. If the shaded square encloses an area of 8, then what is the area enclosed by one of the ellipses?



A. $8\pi\sqrt{2}$ B. 4π C. $4\pi\sqrt{2}$ D. 8π E. NOTA

23. For three distinct parabolas, find the maximum number of points that are on at least two of them.

A. 10 B. 11 C. 12 D. 13 E. NOTA

24. Find the area of the triangle with vertices $A(1, 0, -1), B(5 - 2\ 0), C(2, -1, 3)$.

A. $\sqrt{310}$ B. $\frac{\sqrt{310}}{2}$ C. $\sqrt{374}$ D. $\frac{\sqrt{374}}{2}$ E. NOTA

25. An ant crawls along the parametric curve $x = \cos^2 t\,, y = \cos t \sin t$ for $0 \le t \le 2\pi$. What is the total distance the ant crawled?

A. $\frac{\pi}{2}$ B. π C. 2π D. 4π E. NOTA

26. Find the center of the hyperbola $y = \frac{3x+2}{x+4}$.

A. $(-4, 3)$ B. $(4, 3)$ C. $(3, -4)$ D. $(3, 4)$ E. NOTA

27. An ellipse has foci at points $(1, 0)$ and $(10, 0)$, and passes through the point $(16, 8)$. Which of the following is one of the directrixes of the ellipse?

A. $x = 16$ B. $x = 22$ C. $x = 28$ D. $x = 46$ E. NOTA

28. Rectangle $ABCD$ has vertices A and C at the foci of an ellipse whose area is 2019π, and vertices B and D on the ellipse. If the area of rectangle $ABCD$ is 2019, what is its perimeter?

A. $2\sqrt{2019}$ B. $2\sqrt{4038}$ C. $4\sqrt{4038}$ D. $4\sqrt{2019}$ E. NOTA

29. The set of points z in the Argand plane that satisfy $|z - i + 1| + |z - 3i + 4| = 1$ represents a…

A. Noncircular Ellipse B. Parabola C. Hyperbola D. Circle E. NOTA

30. Which of the following is/are true regarding non-degenerate conics?

I. A hyperbola can be a function.

II. An ellipse can be a function.

III. A parabola can be a function.

A. I B. III C. I, III D. I, II, III E. NOTA