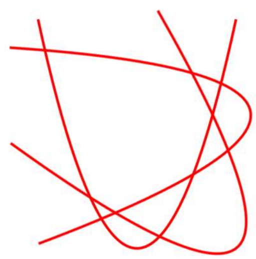
1. B $ab\pi = 2019 \cdot 2\pi = 4038\pi$
2. B Discriminate is $b^2 - 4ac = 1 - 4 = -3$
3. D Rewrite to $4 \cdot \frac{3}{2}\left(y - \frac{1}{3}\right) = x^2$. Thus directrix is $y = \frac{1}{3} - \frac{3}{2} = -\frac{7}{6}$
4. E $c = \sqrt{54} = 2\sqrt{13}, center: (1, -3) \to (1 \pm 2\sqrt{13}, -3)$
5. D $\frac{(x-1)^2}{6^2} - \frac{(y+3)^2}{4^2} = 1. y + 3 = \pm\frac{2}{3}(x - 1) \to y = -\frac{2}{3}x - \frac{7}{3}$
6. A $\frac{(x-1)^2}{6^2} - \frac{(y+3)^2}{4^2} = 1 \to c = 2\sqrt{13} \to \frac{c}{a} = \frac{\sqrt{13}}{3}$
7. D From previously $c = 2\sqrt{13} \to 2c = 4\sqrt{13}$. Length of latus rectum is $\frac{2b^2}{a} = \frac{16}{3}$ so area is $\frac{64\sqrt{13}}{3}$
8. B Set $x = 0 \to -2 - 4y + y^2 = 0 \to 2 \pm \sqrt{6} \mid y = 0, -2 + 4x + 5x^2 = 0 \to -\frac{2}{5} \pm \frac{\sqrt{14}}{5}$.
   $5x_1y_1 = \left(-2 - \sqrt{14}\right)\left(2 - \sqrt{6}\right) = -4 + 2\sqrt{6} - 2\sqrt{14} + 2\sqrt{21}$
9. A $\frac{B}{A-C} = \frac{2}{5-1} = \frac{1}{2}$
10. B $a(x-h)^2 + b(x-h)(y-k) + c(y-k)^2 = f$ which expands to $-f + ah^2 + bhk + ck^2 - 2ahx - bkx + ax^2 - bhy - 2cky + bxy + cy^2 = -2 + 4x + 5x^2 - 4y + 2xy + y^2 \to -f + ah^2 + bhk + ck^2 = -2, -2ah - bk = 4, a = 5, -4 = -bh - 2ck, b = 2, c = 1. \to -10h - 2k = 4, -4 = -2h - 2k \to (h, k) = (-1, 3)$
11. A $-f + ah^2 + bhk + ck^2 = -2 \to -f + 8 = -2 \to f = 10 \to A = \frac{2\pi i}{\frac{1}{f}\sqrt{b^2 - 4ac}} = \frac{20\pi i}{\sqrt{4-20}} = 5\pi$
12. D $r = \frac{1}{2019}\left(ie^{5i\theta} - ie^{-5i\theta}\right)\left(e^{-5i\theta} + e^{5i\theta}\right) = -4\sin(5\theta)\cos(5\theta) = -2\sin(10\theta) \to 20\, petals$
13. A Rewrite into $(x-y)^2 - 2(x-y) + 1 \to (x - y - 1)^2 = 0 \to x - y - 1 = 0 \to r = \frac{1}{\cos\theta - \sin\theta} \to$ $sum\ to\ product \to \frac{\sqrt{2}}{2\sin\left(\frac{\pi}{4} - \theta\right)}$
14. B $b^2 - 4ac = 2^2 \cdot 2019!^2 - 4 \cdot 2019! \cdot 2019! = 0 \to parabola \to eccentricity = 1$
15. A $\frac{eccentricity \cdot d}{1 \pm eccentricity \cdot \cos\theta} \to \frac{\pi^\pi / e^2}{1 - 1\cos(\theta)} \to eccentricity = 1 \to parabola$
16. B Since 3 is between 2 and 4, the limacon is dimpled.
17. C There are many ways to prove this answer, but the simplest to understand is to plug in $a = 0, a = 1 \to q = 2p^2, q = 2p^2 + p + 3 \to p = -3, q = 18 \to p + q = 15$
18. D $4x^2 + y^2 + 8x - 2y + 1 = 0 \to 4(x+1)^2 + (y-1)^2 = 4$. Plug in a and solve for y to get $y = 1 \pm \sqrt{4 - 4(x+1)^2}$. Maximizing the distance squared to $(-2,1)$ gives
   $Max\left((a+2)^2 + \left(1 \pm \sqrt{4 - 4(a+1)^2} - 1\right)^2\right) = Max\left(a^2 + 4a + 4 + 4 - 4(a^2 + 2a + 1)\right) =$
   $Max(-3a^2 - 4a + 4)$. This is a parabola so the max occurs at $-\frac{b}{2a} = -\frac{4}{6} = -\frac{2}{3}$
19. D Both circles have the same y coordinate as their centers so the sum of the slopes has to be 0.
20. A The point (-3,0) is on the second line. Thus, distance is $\left|\frac{-2\cdot 3 - 1}{\sqrt{4+9}}\right| = \frac{7}{\sqrt{13}}$
21. B The vector created by the two points that make the shortest distance will be perpendicular to the two vectors. Hence, find the cross product $\left\|\begin{matrix} i & j & k \\ 2 & -1 & 3 \\ 3 & 2 & 1 \end{matrix}\right\| = -7i + 7j + 7k$. Assume $m(x), n(y)$ gives the points that make the shortest line. Thus, $m(x) - m(y) = z\langle -1, 1, 1\rangle$ for some constant z. This gives the system of equations

$3y-2x+3=-z, 2y+x-1=z, y-3x-2=z \to x=-\frac{1}{7}, y=-\frac{3}{7}, z=-2$. Thus, the distance is $z\langle -1,1,1\rangle = \langle 2,-2,-2\rangle \to$ magnitude $= 2\sqrt{3}$

22. C $c=2, b=2, a^2-b^2=c^2 \to a^2=8 \to a=2\sqrt{2}, area = ab\pi = 4\sqrt{2}$

23. C Note that two distinct parabolas intersect in at most 4 points, which is not difficult to see by drawing examples. Given three parabolas, each pair intersects in at most 4 points, for at
most $4 \cdot 3 = 12$ points of intersection in total. It is easy to draw an example achieving this maximum, for example, by slanting the parabolas at different angles.



This shows 10 intersection with the other two intersections not shown, but one can see if you continue extending the parabolas there will be 12.

24. E Use vectors $\overrightarrow{AB}, \overrightarrow{AC}$ to get $\left\|\begin{vmatrix} i & j & k \\ 4 & -2 & 1 \\ 1 & -1 & 4 \end{vmatrix}\right\| = -7i - 15j - 2k$

Magnitude is $\sqrt{49+225+4} = \sqrt{278}$. $Area\ is\ half\ that = \frac{\sqrt{278}}{2}$

25. C The parametric curve is equivalent to the polar graph $r=\cos\theta$, where $\theta = t$. This is a circle with diameter of 1. For $0 \le t \le 2\pi$, the ant would go around the circle twice, for a total distance of 2π.

26. A The center of a hyperbola is the intersection of its asymptotes. For the hyperbola given, the asymptotes are $x=-4$ and $y=3$.

27. D The distance between the foci is 9, so $c=4.5$. The distances from $(16,8)$ to the two foci are 17 and 10. So the sum of distance from a point on the ellipse to the two foci is 27, or $a=13.5$. Therefore, the eccentricity is $\frac{1}{3}$, and the vertices are $(-8,0)$ and $(19,0)$, the latter is the one relevant to the answer choices. The distance from $(19,0)$ to $(10,0)$ is 9, which must be $\frac{1}{3}$ the distance from $(19,0)$ to the directrix corresponding to $(10,0)$, so the directrix is $x=46$.

28. C $ab=2019$. If the sides of the rectangle are x, y, then $xy=2019$. The distance between the foci is $2\sqrt{a^2-b^2}$. Using Pythagorean theorem, $x^2+y^2=4a^2-4b^2$.
Also, $x+y=2a \to x^2+2xy+y^2=4a^2 \to 4a^2+4b^2+4038=4a^2 \to$
$b=\sqrt{\frac{2019}{2}}, a=\sqrt{2\cdot 2019}$. Area $=2(x+y)=4a=4\sqrt{4038}$

29. E Although this may appear to be a ellipse, this equation has no solution since the sum of the distances from point z and $(-1,1)$ and $(-4,3)$ is always a constant 1. However, notice that the distance between the foci is greater than 1 at $\sqrt{13}$ meaning that no solution exists.

30. C I. True $xy=1$ is a function. II. False trivially. III. True trivially.