Alpha Ciphering nationals 2019 solutions

0- $\left(\cos^2 x + \sin^2 x\right)\left(\cos^2 x - \sin^2 x\right) = \cos 2x \rightarrow \dfrac{2\pi}{b} = \pi$. 1

1. $_{30}C_2 = 435 \rightarrow 435 - 15 - _{15}C_2 = 435 - 15 - 105 = 315$

2. $\dfrac{6\sqrt{2+\sqrt{3}}}{\sqrt{2}+\sqrt{6}} = x \rightarrow x = x^2 = \dfrac{36\left(2+\sqrt{3}\right)}{\left(2+6+2\sqrt{12}\right)2+\sqrt{3}}$

$\dfrac{36\left(2+\sqrt{3}\right)}{\left(8+4\sqrt{3}\right)} = 9 \rightarrow x = 3$

3.
$5 - 9 = 5$
$50 - 99 = 50$
$500 - 999 = 500 \rightarrow 2019 - 555 = 1464 \rightarrow$
$5000 + 1464 - 1 = 6463$

4. The acute angle formed at 2:45 is 172.5 degrees which is 23/48 of a full circle. Multiply this by 8π and you get $\dfrac{23\pi}{6}$. So J is 23

5.

rate	time	distance
48	d/192	d/4
r	3d/4r	3d/4
64	d/64	d

$\dfrac{d}{192} + \dfrac{3d}{4r} = \dfrac{d}{64} \rightarrow \dfrac{1}{192} + \dfrac{3}{4r} = \dfrac{1}{64}$

$\dfrac{3}{4r} = \dfrac{1}{96} \rightarrow 4r = 3 \bullet 96 \rightarrow r = 72$

6. $\left(2^9 + 1\right)^2 - 2(2^9) = \left(2^9 + 1 - 2^5\right)\left(2^9 + 1 + 2^5\right) = 481 \bullet 545 = 13 \bullet 37 \bullet 5 \bullet 109$
$13 + 5 + 37 + 109 = 164$

7. If 3rd and 5th are the same then 1st and 7 are as well. This means the exponent is 6. The 4th term is: $_6C_3L^3\left(-U\right)^3 \to -20$

8. $\dfrac{4500\,rev}{min}\bullet\dfrac{1\,min}{60\,\sec}\bullet\dfrac{2\pi\left(\dfrac{1}{6}\right)}{1\,rev}=\dfrac{25\pi\,ft}{\sec}\to L=25$

9.
$$\left(\sin^2 x+\cos^2 x\right)^2=1=\sin^4 x+2\sin^2 x\cos^2 x+\cos^4 x$$
$$\sin^4 x+\cos^4 x=1-2\sin^2 x\cos^2 x$$
$$2\sin x\cos x=\frac{24}{25}\to 4\sin^2 x\cos^2 x=\frac{576}{625}$$
$$2\sin^2 x\cos^2 x=\frac{288}{625}\to 1-\frac{288}{625}=\frac{337}{625}\to 337$$

10.
$$16x=\left(2x\right)^{\log_2 x}\to 16x=\left(2^{\log_2 x}\right)\left(x^{\log_2 x}\right)$$
$$16=\left(x^{\log_2 x}\right)\to 4=\left(\log_2 x\right)^2\to\log_2 x=\pm 2$$
$$x=2^2,2^{-2}\to\frac{17}{4}\to 21$$

11. Solve for the determinant first ot make sure it is invertible. The determinant is 3. Go to the 3rd row/2md column to get the cofactor entry which is 3. 3 divided by 3 is 1

12. $\dfrac{2015}{5}\to 403\to\dfrac{403}{5}\to 80\to\dfrac{80}{5}\to 16\to\dfrac{16}{5}\to 3$. You do not pick up another zero until you
$403+80+16+3=502$
get to 2020. Therefore 2019 is the largest number.   2019

Answers:

0. 1

1. 315

2. 3

3. 6463

4. 23

5. 72

6. 164

7. -20

8. 25

9. 337

10. 21

11. 1

12. 2019