You may make the following assumptions unless otherwise stated.

- Coins are fair and cannot land on their sides.
- All dice are fair and have six sides, numbered 1 through 6.
- Decks of cards are standard and have 52 cards each. There are no jokers.
- All random numbers, effects, etc. are truly random.

The test will follow the following variable conventions unless otherwise stated.

- All uppercase letter variables are positive integers.
- All fractions containing uppercase letter variables are in lowest terms.

~~~~~~~~ *Good luck, and have fun!* ~~~~~~~~

1) A card is randomly selected from a deck. If the probability that it is neither a club nor a face card is equal to $\frac{A}{B}$, find $A+B$. (Check directions for restrictions on A and B.)

(A) 37 (C) 77 (E) NOTA
(B) 41 (D) 79

2) Find the sum of the digits of $10!+100$.

(A) 26 (C) 28 (E) NOTA
(B) 27 (D) 29

3) A bag contains 3 red, 4 blue, and 3 green balls. Two balls are drawn randomly without replacement. Find the probability that either of the balls drawn is blue.

(A) $\frac{16}{25}$ (C) $\frac{7}{9}$ (E) NOTA
(B) $\frac{2}{3}$ (D) $\frac{5}{6}$

4) Find the constant term in the expansion of $\left(x+2-\frac{1}{x^2}\right)^5$.

(A) -88 (C) 32 (E) NOTA
(B) -72 (D) 48

5) Find the probability that a randomly chosen positive factor of 1000 is also a factor of 320.

(A) $\frac{3}{8}$ (C) $\frac{1}{2}$ (E) NOTA
(B) $\frac{7}{16}$ (D) $\frac{5}{8}$

6) If $p(\alpha)=0.3$, $p(\beta)=0.6$, and $p(\alpha\cup\beta)=0.7$, find $p(\alpha|\beta)$.

(A) $\frac{1}{7}$ (C) $\frac{1}{4}$ (E) NOTA
(B) $\frac{1}{6}$ (D) $\frac{1}{3}$

7) If all of the distinct permutations of the letters in ARMADA are written in alphabetical order, in what position is DAMARA?

(A) 68 (C) 70 (E) NOTA
(B) 69 (D) 71
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8) Three dice are tossed. Given that they have a sum of 8, find the probability that one of them is a 4.

(A) $\frac{4}{39}$
(B) $\frac{3}{7}$
(C) $\frac{5}{11}$
(D) $\frac{6}{13}$
(E) NOTA

9) If the sum of the squares of the reciprocals of all positive integers n such that $\frac{1}{n}$ has a terminating decimal representation is equal to $\frac{P}{Q}$, find $P + Q$.

(A) 3
(B) 17
(C) 25
(D) 43
(E) NOTA

10) Find the number of distinct permutations of the letters in the word MISSISSIPPI that are palindromes (read the same forwards and backwards).

(A) 15
(B) 20
(C) 40
(D) 60
(E) NOTA

For questions 11-13, use the following information.

In a game of *Hearthstone*, two players on opposing sides of a board play minions with certain amounts of health, measured in "hit points" (hp). If a minion takes enough damage that it reaches 0 hp, it instantly dies and is removed from the board.

The spell Minivolcano repeats the following process four times: randomly select a minion on the board, and deal 1 damage to it. Each of the four damage-dealings is called a "hit." It is possible for a minion to be hit multiple times. For example, if there is only a single 4 hp minion on the board, it is guaranteed to be hit four times and die.

11) Trevor casts Minivolcano on a board where he has a minion with 2 hp and his opponent has a minion with 3 hp. Find the probability that the minion with 2 hp dies on exactly the fourth hit of Minivolcano.

(A) $\frac{3}{16}$
(B) $\frac{1}{4}$
(C) $\frac{3}{10}$
(D) $\frac{5}{16}$
(E) NOTA

12) Radleigh casts Minivolcano on a board where he has a minion with 2 hp and his opponent has a minion with 3 hp. Find the probability that the minion with 2 hp dies.

(A) $\frac{3}{5}$
(B) $\frac{11}{16}$
(C) $\frac{3}{4}$
(D) $\frac{13}{16}$
(E) NOTA

13) Richard casts Minivolcano on a board where he has a minion with 2 hp and his opponent has three minions with 1 hp. Find the probability that the minion with 2 hp dies.

(A) $\frac{13}{32}$
(B) $\frac{17}{40}$
(C) $\frac{23}{48}$
(D) The same answer as Question 12
(E) NOTA

14) Each of the 380 1-by-1 squares in a 20-by-19 rectangular grid is drawn. How many rectangles of any size are present in this figure?

(A) 190
(B) 380
(C) $\binom{20}{2}\binom{19}{2}$
(D) $\binom{21}{2}\binom{20}{2}$
(E) NOTA

15) Stirling's approximation says that as n grows large, $n!$ will be asymptotically equivalent to $\sqrt{2\pi n}\left(\frac{n}{e}\right)^n$. By this approximation, $\binom{2n}{n}$ is asymptotically equivalent to $\frac{A^n}{n^{B/8}\sqrt{\pi}}$, find $A + B$.

(A) 8
(B) 10
(C) 12
(D) 16
(E) NOTA

16) The least common multiple of two distinct positive integers α and β is 36. Find the number of ordered pairs (α, β), given that $\alpha > \beta$.

(A) 4
(B) 8
(C) 12
(D) 14
(E) NOTA

17) Connor's calculus class has 12 boys and 8 girls. Arnav's calculus class has 9 boys and 16 girls. A class is randomly chosen, and then a student is randomly selected from the chosen class. Given that the student is a boy, find the probability that he is in Arnav's class.

(A) $\frac{1}{3}$
(B) $\frac{3}{8}$
(C) $\frac{3}{7}$
(D) $\frac{7}{15}$
(E) NOTA

18) Jackson and Couper are playing a game, where the first person to flip heads with an unfair coin wins (if one of them flips tails, they pass the coin to the other person). Given that Jackson has a 62.5% chance of winning if he goes first, find the probability that any given flip of the coin results in heads.

(A) $\frac{2}{7}$
(B) $\frac{1}{3}$
(C) $\frac{2}{5}$
(D) $\frac{1}{2}$
(E) NOTA

19) 1972 IMO Problem 1 read: *Prove that from a set of ten distinct two-digit numbers (in the decimal system), it is possible to select two disjoint subsets whose members have the same sum.* Note that the two sets cannot be empty and the numbers cannot have leading zeroes.

There are $2^{10} - 2 = 1022$ subsets of the set of two-digit numbers. The minimum total sum of the numbers in one of the subsets is 10, and the maximum total sum is 855. This means there are $855 - 10 + 1 = 846$ possible values for the sum of the elements in one of the subsets. Since the number of subsets exceeds the number of possible sums, at least two of the subsets must have the same sum. This is an application of which of the following?

(A) Double Counting
(B) Method of Distinguished Element
(C) Pigeonhole Principle
(D) Principle of Inclusion and Exclusion
(E) NOTA

20) How many nonzero terms are in the expansion of $(4x^3 + 3x^2 + 2x + 1)^6$?

(A) 18
(B) 19
(C) 84
(D) 85
(E) NOTA

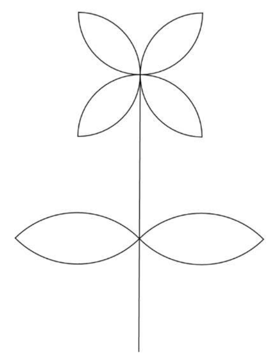
21) Find the maximum integer value of κ such that $\frac{(101^3)!}{(101!)^\kappa}$ is an integer.

(A) 101^2
(B) $101^2 + 1$
(C) $101^2 + 101$
(D) $101^2 + 101 + 1$
(E) NOTA

22) Tyger is playing Dungeons and Dragons. He rolls a fair 20-sided die to determine how much damage he does to a dragon. If he rolls a 12 or less, he deals damage equal to the number on the die. If he rolls in the range [13,19], he deals damage equal to twice the number on the die. If he crits and rolls a 20, he deals damage equal to five times the number on the die. What is the expected amount of damage that Tyger deals to the dragon?

(A) 20.1
(B) 20.3
(C) 20.5
(D) 20.7
(E) NOTA

23) Paloma the parrot loves drawing flowers. She starts at the center of the petals of the flower, then draws the entire flower in one continuous pencil stroke without lifting her pencil from the paper or retracing her path, finishing at the bottom of stem. How many different paths can she take to draw the following flower (which has 4 petals and 2 leaves)? No part (other than points) of the flower may be traced over twice.



(A) 48
(B) 768
(C) 1536
(D) 3072
(E) NOTA

24) Given that $\theta \in [0,2\pi]$, find the probability that $\cos x + \cos 2x \geq 0$.

(A) $\frac{1}{3}$
(B) $\frac{1}{2}$
(C) $\frac{2}{3}$
(D) $\frac{5}{6}$
(E) NOTA

25) A random integer a is chosen from the range $[-10,10]$. Find the probability that the polynomial $x^2 + ax + a + \frac{5}{4}$ has two distinct real roots.

(A) $\frac{4}{7}$
(B) $\frac{2}{3}$
(C) $\frac{16}{21}$
(D) $\frac{3}{4}$
(E) NOTA

26) Steven and Melissa both arrive at a bus stop between 4:00PM and 6:00PM. Steven waits for 30 consecutive minutes, and Melissa waits at the bus stop for 60 consecutive minutes. Given that Steven and Melissa arrive at random times in the range independent of each other, find the probability that Steven and Melissa meet at the bus stop.

(A) $\frac{7}{16}$ (C) $\frac{17}{32}$ (E) NOTA
(B) $\frac{1}{2}$ (D) $\frac{43}{64}$

27) Three distinct vertices of a regular 600-sided polygon are joined. Find the probability that the triangle formed has a right angle.

(A) $\frac{299}{59900}$ (C) $\frac{3}{599}$ (E) NOTA
(B) $\frac{1}{200}$ (D) $\frac{3}{598}$

28) Mettaton and Undyne are playing a best-of-15 pool tournament. Going into the final rack, the score is tied with 7 wins apiece. Throughout the entire tournament, Undyne has never had fewer wins than Mettaton. In how many ways could Mettaton's and Undyne's wins be distributed over the first 14 games?

(A) 429 (C) 433 (E) NOTA
(B) 431 (D) 435

29) The sum of the digits of a seven-digit number is 59. If the probability that the number is divisible by 11 is equal to $\frac{A}{B}$, find $A+B$.

(A) 12 (C) 23 (E) NOTA
(B) 17 (D) 25

30) Consider the following recursion for $n \geq 1$:

$$\alpha_n = \alpha_{n+1}\alpha_{n-1} - \beta_{n+1}\beta_{n-1}$$
$$\beta_n = \beta_{n+1}\alpha_{n-1} + \alpha_{n+1}\beta_{n-1}$$

It is given that $\alpha_0 = 2, \beta_0 = 7, \alpha_1 = 1$, and $\beta_1 = 8$. If $\alpha_{2019}^2 + \beta_{2019}^2 = \frac{P}{Q}$, find $P+Q$.

Hint: Consider $\alpha_n + i\beta_n$.

(A) 54 (C) 98 (E) NOTA
(B) 66 (D) 118