For all questions, answer choice "(E) NOTA" means that none of the given answers is correct. In addition, $i^2 = -1$. The domain of all functions is assumed to be $\mathbb{C}$ unless stated otherwise. Unless instructed otherwise, assume the principal values for arguments of complex numbers. Good luck and have fun!

1. What is $(1+i)^{20} + (1-i)^{18}$?
(A) $2048 - 1024i$ (B) $-1024 - 512i$ (C) $1024 + 512i$ (D) $-1024 + 1024i$ (E) NOTA

2. What is $\frac{3-2i}{1+i} + \frac{2+i}{4+5i}$?
(A) $\frac{67-217i}{82}$ (B) $\frac{48-27i}{82}$ (C) $\frac{7+2i}{82}$ (D) $\frac{23-38i}{82}$ (E) NOTA

3. What is $|(3-4i)(5+12i)(24+7i)|$?
(A) 1325 (B) 1525 (C) 1425 (D) 1625 (E) NOTA

4. What is $\left(1+i\sqrt{3}\right)^{10}$?
(A) 1024 (B) $-1024 - 1024i\sqrt{3}$ (C) $512 - 512i\sqrt{3}$ (D) $-512 - 512i\sqrt{3}$ (E) NOTA

5. What is the resulting vector when $\langle 3, 5\rangle$ is rotated by $\frac{\pi}{4}$ counterclockwise?
(A) $\langle -\sqrt{2}, 4\sqrt{2}\rangle$ (B) $\langle -3\sqrt{2}, 5\sqrt{2}\rangle$ (C) $\langle -\sqrt{2}, 2\sqrt{2}\rangle$ (D) $\langle -2\sqrt{2}, 8\sqrt{2}\rangle$ (E) NOTA

6. If $2z_1 + z_2 = 5 + 4i$ and $3z_1 - 2z_2 = 4 - i$, then what is the value of $|z_1 + z_2|$?
(A) $2\sqrt{3}$ (B) $3\sqrt{2}$ (C) $\sqrt{6}$ (D) 6 (E) NOTA

7. What is the polar form of $\frac{12}{3-i\sqrt{3}}$?
(A) $4\text{cis}\left(\frac{\pi}{3}\right)$ (B) $2\sqrt{3}\text{cis}\left(\frac{\pi}{6}\right)$ (C) $3\text{cis}\left(\frac{2\pi}{3}\right)$ (D) $4\text{cis}(\frac{5\pi}{6})$ (E) NOTA

8. Sequence a_n is given by $a_0 = i$ and $a_{n+1} = a_n^2 + a_n$ for $n \geq 0$. Find a_{100}.
(A) $-1 + i$ (B) i (C) $1 + i$ (D) $-1 - i$ (E) NOTA

9. $f(x)$ is a quadratic equation with real coefficients and leading coefficient 1. Given that $f(6-2i)=0$, what is the value of $f(8)$?

(A) 4 (B) 2 (C) 8 (D) 16 (E) NOTA

10. What is the graph formed by the set of complex numbers on the Argand plane that satisfy the equation $z\bar{z}=1$?

(A) Line (B) Parabola (C) Hyperbola (D) Circle (E) NOTA

11. What is a_{2018} if $a_0=a_1=i$ and $a_n=a_{n-1}a_{n-2}$ for $n\geq 2$?

(A) i (B) -1 (C) $-i$ (D) 1 (E) NOTA

12. For how many integers n does the graph of $f(x)=x^2+nx+n$ not intersect the x-axis?

(A) 2 (B) 3 (C) 4 (D) 5 (E) NOTA

13. z is a complex number with integral real and imaginary parts. Which of the following is not a possible value of $z\cdot\bar{z}$?

(A) 2018 (B) 2020 (C) 2017 (D) 2019 (E) NOTA

14. Given that $z^2+8-6i=0$, what is the value of $|z-\bar{z}|+|z+\bar{z}|$?

(A) 4 (B) 8 (C) 12 (D) 16 (E) NOTA

15. What is the area of the polygon on the complex plane with vertices that are the roots of $f(x)=4x^4-16x^3+24x^2-16x+13$?

(A) 9 (B) 7 (C) 5 (D) 3 (E) NOTA

16. What is the value of $\sqrt{3+i2\sqrt{3}\sqrt{3+i2\sqrt{3}\sqrt{3+\ldots}}}$?

(A) $i\sqrt{7}$ (B) $\frac{3}{4}+\frac{i\sqrt{3}}{5}$ (C) $\frac{3i}{5}$ (D) 1 (E) NOTA

17. What is $\left(\sqrt{i}\right)^{i}$?

(A) $e^{-\frac{\pi}{2}}$ (B) $e^{-\frac{\pi}{4}}$ (C) $e^{\frac{\pi}{4}}$ (D) $e^{\frac{\pi}{2}}$ (E) NOTA

18. What is $\prod_{n=1}^{360}\left(\text{cis}(n^\circ)\right)^{360-n}$?

(A) i (B) $\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}$ (C) $\frac{1}{2}+\frac{i\sqrt{3}}{2}$ (D) $\frac{\sqrt{3}}{2}-\frac{i}{2}$ (E) NOTA

19. Let a_n be a geometric sequence with $a_0 = 1$ and ratio $r = \text{cis}(k)$. For how many $0^\circ \le k \le 360^\circ$ is it true that the smallest $m > 0$ such that $a_m = 1$ is $m = 360$?

(A) 120 (B) 150 (C) 180 (D) 210 (E) NOTA

20. Mr. Lu is walking along the complex plane according to the following rules: he starts at the origin facing towards the positive real axis, then for every n^{th} move, he moves n units forward and then turns $\frac{\pi}{2}$ radians to the left. After 2018 moves, where is Mr. Lu on the complex plane?

(A) $-1009-1009i$ (B) $1009+1010i$ (C) $2018-2019i$ (D) $2017+2017i$ (E) NOTA

21. What is $\cos(72^\circ)+\cos(144^\circ)$?

(A) 0 (B) $-\frac{\sqrt{3}}{6}$ (C) $-\frac{1}{2}$ (D) $-\frac{\sqrt{5}}{4}$ (E) NOTA

22. What is $\cos(4\theta)$ when expressed as a function of $\cos(\theta)$ only?
(A) $\cos^4(\theta) - \cos^2(\theta)$ (B) $8\cos^4(\theta) - 8\cos^2(\theta) + 1$

(C) $4\cos^4(\theta) - 1$ (D) $\cos^4(\theta)$ (E) NOTA

23. What quadrant is $(2017 - 2018i)^{50}$ in?
(A) IV (B) III (C) II (D) I (E) NOTA

24. What is the distance between the foci of the conic $13x^2 + 10xy + 13y^2 = 72$?
(A) 2 (B) $2\sqrt{3}$ (C) $2\sqrt{5}$ (D) $2\sqrt{7}$ (E) NOTA

25. Zhao is at $P_0 = 13 + 84i$ and wishes to walk back to his home at the origin. He takes a puzzling route P_0P_1, P_1P_2, P_2P_3, ..., where $P_n = \left(\frac{1}{2} + \frac{1}{2}i\right) P_{n-1}$. What is the distance that Zhao must walk before he reaches his home?
(A) $\frac{85\sqrt{2}}{2}$ (B) $85\sqrt{3} + 85$ (C) $85\sqrt{2} + 85$ (D) $85\sqrt{6} - 85$ (E) NOTA

26. What is the coefficient of the x^4 term in the expression $(ix + 2)^8$?
(A) 1120 (B) $-560i$ (C) $780i$ (D) -1120 (E) NOTA

27. For how many positive integers n is $|18 + ni|$ an integer?
(A) 2 (B) 3 (C) 4 (D) 5 (E) NOTA

28. Suppose that $P_1 = z, P_2 = 2z^2$, and $P_3 = -3z^3$ are the vertices of an isosceles triangle on the complex plane with equal sides $\overline{P_1P_2}$ and $\overline{P_2P_3}$. The graph of all such z forms a closed shape with area A. What is $\lfloor A \rfloor$?
(A) 8 (B) 1 (C) 7 (D) 3 (E) NOTA

29. Function $f: [-1,1) \to \mathbb{C}$ has the properties that it is one-to-one, $f(-1) = -1$, and $f(a)f(b) = f(a+b)$ for all $a, b, a+b \in [-1,1)$. What is $f\left(\frac{2}{3}\right) + f(-\frac{2}{3})$?

(A) 1 (B) i (C) -1 (D) $-i$ (E) NOTA

30. Ben and David are playing a game in which they take turns selecting four numbers k_1, k_2, k_3, k_4 from the set $\{0,1\}$, randomly and with replacement. They then each determine their value $|i^{k_1} + i^{k_2} + i^{k_3} + i^{k_4}|$. If their magnitudes are equal, then Ben wins! What is the probability that Ben wins?

(A) $\frac{3}{8}$ (B) $\frac{25}{64}$ (C) $\frac{13}{32}$ (D) $\frac{7}{16}$ (E) NOTA