For all questions, the answer choice "(E) NOTA" means none of the above answers is correct. Unless told otherwise, assume the domain of all questions to be $\mathbb{R}$. Good luck and have fun!

1. On Ellen's farm, there live a certain number of giraffe friends and human friends. Ellen and her 19 friends live on her farm, and there are 56 legs on her farm in total. How many giraffes are on Ellen's farm?

(A) 5 (B) 6 (C) 7 (D) 8 (E) NOTA

2. How many integers x satisfy the inequality $\big||x| - 2\big| \leq 2$?

(A) 7 (B) 8 (C) 9 (D) 10 (E) NOTA

3. How many solutions does the equation $5x + 3y = 100$ have over positive integers (x, y)?

(A) 6 (B) 7 (C) 8 (D) 9 (E) NOTA

4. What is the sum of all $0 \leq \theta \leq \pi$ such that $\sin(2\theta) = \tan(\theta)$?

(A) 2π (B) $\frac{5\pi}{2}$ (C) 3π (D) $\frac{7\pi}{2}$ (E) NOTA

5. If $2\log_2(x+3) - \log_2 x = 4$, what is the sum of the solutions for x?

(A) 10 (B) 16 (C) 22 (D) 28 (E) NOTA

6. Given $\cos\theta + \sin\theta = \frac{5}{4}$ and $0 < \theta < \frac{\pi}{4}$, what is $\cos(2\theta)$?

(A) $\frac{5\sqrt{7}}{16}$ (B) $\frac{9}{16}$ (C) $\frac{3}{4}$ (D) $\frac{\sqrt{7}}{4}$ (E) NOTA

7. Given α, β, and γ are three angles of a triangle with $\frac{\pi}{2} < \gamma < \pi$ and $\sin(\alpha + \beta) = \frac{4}{5}$, what is the value of $\cos\gamma$?

(A) $-\frac{4}{5}$ (B) $\frac{\sqrt{3}}{4}$ (C) $\frac{\sqrt{5}}{5}$ (D) $\frac{3}{5}$ (E) NOTA

8. Sanika and Beverly are rowing along a river. It takes them 10 hours to row upstream from their starting point, and it takes them 6 hours to row downstream returning to the same point. How long would it take for Sanika and Beverly to row the same distance (one way) in a lake without any current?

(A) $\frac{13}{2}$ (B) $\frac{15}{2}$ (C) $\frac{23}{3}$ (D) $\frac{19}{2}$ (E) NOTA

9. For how many ordered triples of odd positive integers (a, b, c) is $a + 2b + 2c = 81$?

(A) 210 (B) 231 (C) 330 (D) 364 (E) NOTA

10. Given $x, y > 0$,

$$3\lfloor x \rfloor + y = 10.4$$
$$x + 2\lfloor y \rfloor = 5.6$$

what is $x + y$?

(A) 6 (B) 7 (C) 8 (D) 9 (E) NOTA

11. What is the point that results from the reflection of the point $(6,9,0)$ across the plane $x + 4y + 2z = 0$?

(A) $(4,1,-1)$ (B) $(3,-2,-3)$ (C) $(2,-7,-8)$ (D) $(1,-5,-9)$ (E) NOTA

12. Ben's favorite equation is the equation $\big|\big||x-3|-3\big|-3\big| = 3$. How many solutions are there to Ben's favorite equation?

(A) 2 (B) 4 (C) 6 (D) 8 (E) NOTA

13. If $\dfrac{\cos^2\theta}{2+\frac{\cos^2\theta}{2+\cdots}} = \dfrac{1}{4}$ for some $0 < \theta < \frac{\pi}{2}$, then what is $\tan\theta$?

(A) $\dfrac{\sqrt{7}}{3}$ (B) $\dfrac{\sqrt{6}}{4}$ (C) $\dfrac{\sqrt{5}}{5}$ (D) $\dfrac{1}{3}$ (E) NOTA

14. What is the graph represented by the equation $x^2 + 6xy + 9y^2 = 9$?

(A) Hyperbola (B) Circle (C) Ellipse (D) Parabola (E) NOTA

15. The points (1,7,2), (7,2,9), and (2,9,1) lie on a plane with the equation $Ax + By + Cz + D = 0$, where A, B, C, and D are integers with $\gcd(A, B, C, D) = 1$. What is the value of $A^2 + B^2 + C^2$?

(A) 513 (B) 539 (C) 604 (D) 675 (E) NOTA

16. What is the shortest distance between the sphere with equation $(x-2)^2 + (y+1)^2 + (z-3)^2 = 2$ and the plane with equation $3x + 4y + 5z = 2$?

(A) $\dfrac{\sqrt{2}}{5}$ (B) $\dfrac{\sqrt{2}}{4}$ (C) $\dfrac{\sqrt{2}}{3}$ (D) $\dfrac{\sqrt{2}}{2}$ (E) NOTA

17. For what value of n will the system of equations

$$x + y + z = 192$$
$$2x + 6y + 3z = 393$$
$$4x + 2y + nz = 954$$

have no solutions?

(A) $\dfrac{5}{2}$ (B) 3 (C) $\dfrac{7}{2}$ (D) 4 (E) NOTA

18. What is the minimum value of $\dfrac{3x^2-6x+7}{x-1}$ for $x > 1$?

(A) $4\sqrt{3}$ (B) $5\sqrt{2}$ (C) 6 (D) $2\sqrt{5}$ (E) NOTA

19. Given $xy = 9$, what is the value of y that minimizes the value of the expression $x^3y + xy^2 + 9y$ for $x, y > 0$?

(A) $\sqrt{3}$ (B) $2\sqrt{3}$ (C) $3\sqrt[3]{3}$ (D) $3\sqrt[3]{9}$ (E) NOTA

20. If $\sqrt{x + 4\sqrt{x + \sqrt{x + 4\sqrt{x + \cdots}}}} = 3$, then what is the value of x?

(A) 3 (B) 4 (C) 5 (D) 6 (E) NOTA

21. What is the value of $\cos 72° + \cos 144°$?

(A) $-\frac{1}{2}$ (B) $-\frac{\sqrt{3}}{2}$ (C) $\frac{\sqrt{2}-\sqrt{6}}{4}$ (D) $-\frac{\sqrt{5}}{5}$ (E) NOTA

22. What is the area of the graph given by $x = 3\cos t$ and $y = 2\sin t$?

(A) 2π (B) 3π (C) 4π (D) 5π (E) NOTA

23. What is the graph formed by the parametric equations $x = 3\tan(t)$ and $y = 2\sec(t)$?

(A) Parabola (B) Hyperbola (C) Ellipse (D) Limaçon (E) NOTA

24. Given that $f(x)$ is a quartic polynomial, and given the following values:

x	10	11	12	13	14
$f(x)$	1	5	11	16	18

What is $f(15)$?

(A) 22 (B) 20 (C) 18 (D) 16 (E) NOTA

25. Given that the solutions of the equation $x^3 - 7x^2 + 12x - 14 = 0$ are p, q, and r, what is the value of $p^3 + q^3 + r^3$?

(A) -28 (B) 83 (C) 104 (D) 133 (E) NOTA

Questions 26-29 relate to finding solutions to a Diophantine equation known as Pell's Equation.

26. A **continued fraction** is a number of the form $[a_0; a_1, a_2, \dots] = a_0 + \frac{1}{a_1 + \frac{1}{a_2 + \cdots}}$. For example, $\frac{11}{7} = [1; 1,1,3] = 1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{3}}}$ and $e = [2; 1,2,1, \dots] = 2 + \frac{1}{1 + \frac{1}{2 + \cdots}}$. If $\sqrt{2} = [a_0; a_1, a_2, \dots]$, what is a_1?

(A) 2 (B) 3 (C) 4 (D) 5 (E) NOTA

27. The nth **convergent** of a continued fraction is the expression given by evaluating $[a_0; a_1, a_2, \dots]$ up to the nth term. For example, if $e = [2; 1,2,1, \dots]$, then its 2nd convergent is $2 + \frac{1}{1 + \frac{1}{2}} = \frac{8}{3}$. Given $\pi = [3; 7,15,1, \dots]$, what is the 2nd convergent of π?

(A) $\frac{232}{73}$ (B) $\frac{22}{7}$ (C) $\frac{353}{113}$ (D) $\frac{333}{106}$ (E) NOTA

28. Pell's Equation is a Diophantine equation of the form $x^2 - Ny^2 = 1$. The **fundamental solution** is the solution to a Pell's Equation with positive x, y such that x is as small as possible. It can be found by checking successive convergents $\frac{p}{q}$ of $\sqrt{N}$ for $p^2 - Nq^2 = 1$. Given that $\sqrt{5} = [2; 4,4,4, \dots]$, and the fundamental solution to $x^2 - 5y^2 = 1$ is (x_1, y_1), what is $x_1 + y_1$?

(A) 13 (B) 19 (C) 45 (D) 85 (E) NOTA

29. By Brahmagupta's identity, we can find that $(x_1^2 - Ny_1^2)(x_2^2 - Ny_2^2) = (x_1x_2 - Ny_1y_2)^2 - N(x_1y_2 - x_2y_1)^2$. Thus, given the fundamental solution (x_1, y_1), successively larger positive solutions (x_n, y_n) can be found by the equation $x_n + y_n\sqrt{N} = \left(x_1 + y_1\sqrt{N}\right)^n$. If the 2nd smallest solution to $x^2 - 5y^2 = 1$ is (x_2, y_2), what is $x_2 + y_2$?

(A) 233 (B) 157 (C) 100 (D) 45 (E) NOTA

30. What is the remainder when $x^{80} - 9x^{78} + 10$ is divided by $x^2 - 4x + 3$?

(A) $4x - 2$ (B) $4x + 2$ (C) $2x - 4$ (D) $2x + 4$ (E) NOTA