1. D
2. C
3. A
4. A
5. A
6. A
7. E
8. B
9. A
10. E
11. C
12. B
13. A
14. E
15. B
16. D
17. C
18. A
19. C
20. E
21. A
22. E
23. B
24. D
25. D
26. A
27. D
28. A
29. A
30. A

1. Let a be the number of giraffes on Ellen's farm and let b be the number of humans on Ellen's farm. Then
$$a + b = 20$$
$$4a + 2b = 56$$
Then $4a + 2b - 2(a + b) = 56 - 2(20) \to 2a = 16 \to a = 8$. So there are 8 giraffes on Ellen's farm. $\boxed{D}$

2. $||x| - 2| \leq 2$ implies that $-2 \leq |x| - 2 \leq 2$ which means that $0 \leq |x| \leq 4$. Finally we know that $-4 \leq x \leq 4$. Thus there are a total of 9 values that satisfy this inequality. $\boxed{C}$

3. The solutions enumerated are $(17, 5), (14, 10), \ldots, (2, 30)$. Thus there are 6 solutions to the equation. $\boxed{A}$

4. Subtracting $\tan\theta$ from both sides we get $\sin(2\theta) - \tan\theta = 2\sin\theta\cos\theta - \frac{\sin\theta}{\cos\theta} = \sin\theta(2\cos\theta - \frac{1}{\cos\theta}) = 0$ Then we have either $\sin\theta = 0$ or $2\cos\theta - \frac{1}{\cos\theta} = 0$ Taking the latter case we get $2\cos^2\theta - 1 = 0 \to \cos\theta = \pm\frac{\sqrt{2}}{2}$. Then the angles which satisfy $\sin\theta = 0$, $\cos\theta = \pm\frac{\sqrt{2}}{2}$ are $\theta = 0, \frac{\pi}{4}, \frac{3\pi}{4}, \pi$ and the sum is $0 + \frac{\pi}{4} + \frac{3\pi}{4} + \pi = 2\pi$ $\boxed{A}$

5. Combining the logs on the left side gives $\log_2 \frac{(x+3)^2}{x} = 4$. This means that $\frac{(x+3)^2}{x} = 16 \to (x + 3)^2 = 16x \to x^2 - 10x + 9 = (x - 9)(x - 1) = 0$. Then the sum of solutions for x is $9 + 1 = 10$ $\boxed{A}$

6. Squaring both sides of the equation gives $(\cos\theta + \sin\theta)^2 = \cos^2\theta + 2\cos\theta\sin\theta + \sin^2\theta = 1 + \sin(2\theta) = (\frac{5}{4})^2 \to \sin(2\theta) = \frac{25}{16} - 1 = \frac{9}{16}$. Then using Pythagorean identity, $\cos^2(2\theta) = 1 - \sin^2(2\theta) = 1 - \frac{81}{256} = \frac{175}{256} \to \cos(2\theta) = \frac{5\sqrt{7}}{16}$ $\boxed{A}$

7. Since $\alpha + \beta + \gamma = \pi$, $\sin(\alpha + \beta) = \sin(\pi - \alpha - \beta) = \sin\gamma = \frac{4}{5}$. Then $\cos^2\gamma = 1 - \sin^2\gamma = 1 - \frac{16}{25} = \frac{9}{25} \to \cos\gamma = -\frac{3}{5}$ $\boxed{E}$

8. Let r be the rate that Sanika and Beverly row, let c be the rate of the current, and let x be the length of the river. Then $10(r - c) = x$ and $6(r + c) = x$. Then $2r = (r - c) + (r + c) = \frac{x}{10} + \frac{x}{6} = \frac{4x}{15} \Rightarrow \frac{15r}{2} = x$. Then it will take them $\frac{15}{2}$ hours to row along the river with no current. $\boxed{B}$

9. Since $a, b,$ and$c$ are odd integers we can say $a = 2a_0 + 1, b = 2b_0 + 1, c = 2c_0 + 1$ for non-negative a_0, b_0, c_0. Substituting these in we get $2a_0 + 1 + 2(2b_0 + 1) + 2(2c_0 + 1) = 2a_0 + 4b_0 + 4c_0 + 5 = 81 \to a_0 + 2b_0 + 2c_0 = 38$. Then since 38 and $2b_0 + 2c_0$ are even, it's obvious that a_0 is even. Then we can say $a_0 = 2a_1$ for non-negative a_1. Substituting that in we find that $2a_1 + 2b_0 + 2c_0 = 38 \to a_1 + b_0 + c_0 = 19$. By stars and bars we find that there are $\binom{21}{2}$ non-negative integer solutions (a_1, b_0, c_0). Since every non-negative solution (a_1, b_0, c_0) corresponds to one positive odd solution (a, b, c), there are $\binom{21}{2} = 210$ solutions. $\boxed{A}$

10. Since $\lfloor x \rfloor$ and $\lfloor y \rfloor$ are integers, it is obvious that the decimal parts of x and y are .4 and .6, respectively. Let $x = a + .4$ and $y = b + .6$ where a, b are integers. Then $3a + b = 10$ and $a + 2b = 5$. Then solving for a, b gives $a = 3, b = 1$. Then $x + y = 3.4 + 1.6 = 5$ $\boxed{E}$

11. The normal vector from the plane $x + 4y + 2z = 0$ is $< 1, 4, 2 >$. Then the vector $< 6+t, 9+4t, 2t >$ is perpendicular to the given plane and the point that is on the plane as well as the vector is given by $6+t+4(9+4t)+2(2t) = 42+21t = 0 \to t = -2$. Then the point which is reflected is $(6 + 2(1)(-2), 9 + 2(4)(-2), 2(2)(-2)) = (2, -7, -8)$ $\boxed{C}$

12. $|||x - 3| - 3| - 3| = 3$ means that $||x - 3| - 3| = 0, 6$, then $|x - 3| = -3, 3, 9$, and $x = -6, 0, 6, 12$. Then there are 4 solutions to Ben's equation. $\boxed{B}$

13. Substituting at the point when the expression repeats gives $\frac{\cos^2\theta}{2+\frac{1}{4}} = \frac{\cos^2\theta}{\frac{9}{4}} = \frac{1}{4} \to \cos^2\theta = \frac{9}{16} \to \cos\theta = \frac{3}{4}$. Then $\sin^2\theta = 1 - \cos^2\theta = 1 - \frac{9}{16} = \frac{7}{16} \to \sin\theta = \frac{\sqrt{7}}{4}$. So $\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\sqrt{7}}{3}$ $\boxed{A}$

14. We can factor the left side giving $(x + 3y)^2 = 9$. Then factoring by difference of squares gives $(x + 3y - 3)(x + 3y + 3) = 0$. Then $x + 3y - 3 = 0$ and $x + 3y + 3 = 0$ which is two parallel lines. $\boxed{E}$

15. We can find the normal vector to the plane by taking the cross product of two vectors between the three points. The vector from $(1, 7, 2)$ to $(7, 2, 9)$ is $< 6, -5, 7 >$. The vector from $(1, 7, 2)$ to $(2, 9, 1)$ is $< 1, 2, -1 >$. Then the cross product $< 6, -5, 7 > \times < 1, 2, -1 >$ is $\begin{vmatrix} i & j & k \\ 6 & -5 & 7 \\ 1 & 2 & -1 \end{vmatrix} = -9i+13j+17k$ Then $A^2+B^2+C^2 = (-9)^2+(13)^2+(17)^2 = 81 + 169 + 289 = 539$ $\boxed{B}$

16. The shortest distance can be found by finding the distance between the center of the sphere and the plane and subtracting the radius. The distance between a point (a, b, c) and a plane $Ax + By + Cz = K$ is given by $D = \frac{|Aa+Bb+Cc-K|}{\sqrt{A^2+B^2+C^2}} = \frac{|(3)(2)+(4)(-1)+(5)(3)-2|}{\sqrt{3^2+4^2+5^2}} = \frac{15}{\sqrt{50}} = \frac{3\sqrt{2}}{2}$. The radius of the sphere is $\sqrt{2}$. Thus the shortest distance between the sphere and the plane is $\frac{3\sqrt{2}}{2} - \sqrt{2} = \frac{\sqrt{2}}{2}$ $\boxed{D}$

17. Subtracting 2 times the first equation from the second equation gives $2x + 6y + 3z - 2(x + y + z) = 4y + z = 9$. Subtracting 4 times the first equation from the third equation gives $4x + 2y + nz - 4(x + y + z) = -2y + (n - 4)z = 954 - 768 = 186$. Then $4y + z + 2(-2y + (n - 4)z) = (2n - 7)z = 9 + 2(186) = 381$. If we have $n = \frac{7}{2}$ then we get $0 = 381$ which is obviously false. $\boxed{C}$

18. Dividing through we get $\frac{3x^2-6x+7}{x-1} = 3(x - 1) + \frac{4}{x-1}$. Then by AM-GM we know that $3(x - 1) + \frac{4}{x-1} \geq 2\sqrt{3(x - 1)\frac{4}{(x-1)}} = 2\sqrt{12} = 4\sqrt{3}$ which is achieved at $x = 1 + \frac{2}{\sqrt{3}}$ $\boxed{A}$

19. By using AM-GM we see that $x^3y + xy^2 + 9y \geq 3\sqrt[3]{9x^4y^4} = 3\sqrt[3]{9(9)^4} = 3\sqrt[3]{3^{10}} = 81\sqrt[3]{3}$ which is achieved at $x = \sqrt[3]{9}, y = 3\sqrt[3]{3}$ $\boxed{C}$

20. Substituting at the point at which the expression repeats gives $\sqrt{x + 4\sqrt{x + 3}} = 3 \rightarrow x^2 - 34x + 33 = 0$ which gives $x = 33, 1$. Clearly $x = 1$ satisfies the expression. $\boxed{E}$

21. Note that the 5th roots of unity are $1, \text{cis}\, 72°, \text{cis}\, 144°, \text{cis}\, 216°, \text{cis}\, 288°$. The sum of roots is $1+\text{cis}\, 72°+\text{cis}\, 144°+\text{cis}\, 216°+\text{cis}\, 288° = 0$, which implies that the sum of the real parts of the roots must also be 0. This means that $1+\cos 72°+\cos 144°+\cos 216°+\cos 288° = 0$. Then since cos is an even function we know that $\cos 216° = \cos(-216°) = \cos 144°$ and $\cos 288° = \cos(-288°) = \cos 72°$. Then, $1 + \cos 72° + \cos 144° + \cos 216° + \cos 288° = 1 + 2(\cos 72° + \cos 144°) = 0$. Thus, $\cos 72° + \cos 144° = -\frac{1}{2}$ $\boxed{A}$

22. $1 = \cos^2 t + \sin^2 t = (\frac{x}{3})^2 + (\frac{y}{2})^2 = \frac{x^2}{9} + \frac{y^2}{4}$. This is an ellipse with area $ab\pi = (3)(2)\pi = 6\pi$ $\boxed{E}$

23.

$$x = 3\tan(t) \rightarrow \tan(t) = \frac{x}{3}$$

$$y = 2\sec(t) \rightarrow \sec(t) = \frac{y}{2}$$

$$\sec^2(t) - \tan^2(t) = \frac{y^2}{4} - \frac{x^2}{9} = 1$$

$\boxed{B}$

24. Let n be the value of $f(15)$. Looking at the sequence of differences between the terms $\{1, 5, 11, 16, 18, n\}$ gives us $\{4, 6, 5, 2, n-18\}$. The sequence of second differences is then $\{2, -1, -3, n-20\}$. The sequence of third differences is $\{-3, -2, n-17\}$. The sequence of fourth differences is $\{1, n-15\}$. Since f is a quartic, the fourth differences must be constant. Then $n - 15 = 1 \rightarrow n = 16$ $\boxed{D}$

25. First we know that the sum of the roots $p + q + r = 7$. Additionally, the sum of the roots taken two at a time is $pq + qr + pr = 12$. Now we can find $p^2 + q^2 + r^2 = (p + q + r)^2 - 2(pq + qr + pr) = 7^2 - 2(12) = 25$. Since p, q, r are roots of the equation we know that $p^3 - 7p^2 + 12p - 14 = 0, q^3 - 7q^2 + 12q - 14 = 0, r^3 - 7r^2 + 12r - 14$. Then $p^3 + q^3 + r^3 = 7(p^2 + q^2 + r^2) - 12(p + q + r) + 52 = 7(25) - 12(7) + 42 = 133$ $\boxed{D}$

26. Since $1 < \sqrt{2} < 2$, $a_0 = 1$. Then $2 < \frac{1}{\sqrt{2}-1} < 3$ so $a_1 = 2$ $\boxed{A}$

27. The second convergent of π is given by $3 + \frac{1}{7+\frac{1}{15}} = 3 + \frac{15}{106} = \frac{333}{106}$ $\boxed{D}$

28. The first convergent of $\sqrt{5}$ is $\frac{2}{1}$ which gives $2^2 - 5(1)^2 = -1$ is not a solution. The second convergent of $\sqrt{(5)}$ is $2 + \frac{1}{4} = \frac{9}{4}$ which gives $9^2 - 5(4)^2 = 1$, thus $(9, 4)$ is the fundamental solution and $x_1 + y_1 = 9 + 4 = 13$ $\boxed{A}$

29. $x_2 + y_2\sqrt{N} = (9 + 4\sqrt{5})^2 = 161 + 72\sqrt{5}$. So $x_2 + y_2 = 161 + 72 = 233$ $\boxed{A}$

30. The division algorithm tells us that $x^{80} - 9x^{78} + 10 = (x^2 - 4x + 3)Q(x) + R(x)$ where $Q(x)$ has degree 78 and $R(x)$ has degree of at most 1. Then we know that $x^{80} - 9x^{78} + 10 = (x^2 - 4x + 3)Q(x) + Ax + B$. If we plug in $x = 1, 3$, we see that

$$3^{80} - 9(3^{78}) + 10 = (3^2 - 4(3) + 3)Q(3) + 3A + B$$

$$1^{80} - 9(1^{78}) + 10 = (1^2 - 4(1) + 3)Q(1) + A + B$$

which reduces to the equations

$$10 = 3A + B$$

$$2 = A + B$$

Solving this system gives $A = 4, B = -2$. So our remainder is $4x - 2$ $\boxed{A}$