

For all questions, answer choice "E. NOTA" means none of the above answers are correct.

- Solve for  $x$ :  $\log_3 3x = 3$   
A. 3                      B. 9                      C. 27                      D. 1                      E. NOTA
- What is  $\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n$ ?  
A. 1                      B. 0                      C.  $e$                       D.  $\infty$                       E. NOTA
- What is  $e^{i\theta}$ ?  
A.  $\cos \theta + i \sin \theta$     B.  $i \sin \theta$     C.  $i \cos \theta + \sin \theta$     D.  $\cos \theta - i \sin \theta$     E. NOTA
- Google gives every page on the web a PageRank Score with formula  $PageRank = k \log V$ , where  $V$  is the number of visits the page receives in a day, and  $k$  is a set constant. If Steve's favorite website (penguinonly.com) gets 16,500 visits/day and has a PageRank of  $4k$ , then how many visits per day does Sohan's favorite website (legendsonly.com) receive if it has a PageRank of  $7k$ ?  
A. 16,500,000    B. 28,875    C. 49,500    D. 1,650,000    E. NOTA
- If  $\sqrt{1406 - \sqrt{1406 - \sqrt{1406} \dots}} = x$ , calculate  $x$ .  
A. 38                      B. 37                      C.  $\sqrt{1406}$                       D. 36                      E. NOTA
- Evaluate:  $\frac{1}{\log_2 48} + \frac{1}{\log_4 48} + \frac{1}{\log_6 48} + \frac{1}{\log_8 48} + \frac{1}{\log_{12} 48} + \frac{1}{\log_{24} 48}$   
A. 6                      B. 8                      C. 48                      D. 3                      E. NOTA
- Simplify the following expression, where defined:  $\frac{x^2 \left(y^{\frac{1}{2}}\right)^3 z^5}{x^{-5} y^{\frac{-1}{2}} (z^{10})^{\frac{1}{2}}}$   
A.  $x^{-3}$                       B.  $x^{-3} y z^{10}$                       C.  $x^7 y$                       D.  $x^7 y^2$                       E. NOTA

8. Solve for  $x$ :  $\log_{\sqrt{6}} \begin{vmatrix} 2 & 3 & 2 \\ 2 & 3 & 1 \\ 4 & 9 & 1 \end{vmatrix} = x$

A.  $\frac{1}{2}$

B. 1

C. 2

D. 4

E. NOTA

Questions 9 and 10 pertain to the following prompt:

Qing is running toward Mr. Lu, who is enjoying his bagel. Her velocity is modeled with the equation  $v(t) = -2t^2 + 9t - 4$ , where  $t$  is measured in seconds (s), and  $v(t)$  is measured in feet per second (ft/s).

9. What is the maximum velocity that Qing reaches?

A.  $\frac{9}{4}$

B.  $\frac{49}{8}$

C.  $\frac{9}{2}$

D.  $\frac{49}{4}$

E. NOTA

10. After reaching her maximum velocity, Qing reaches Mr. Lu when her velocity is 0. How long has she been running since reaching that maximum velocity?

A. 4

B. 1.75

C. 2.25

D. 0.5

E. NOTA

11. Find the sum of the solutions for  $x$ :  $-17 \log x + (\log x)^2 + 74 = 2$

A. 17

B. 9

C.  $10^{17}$

D. 8

E. NOTA

12. If A is the product of the roots of the equation  $\sqrt[3]{4001} \cdot x^{\log_{4001} x} = x^8$ , find A.

A. 8

B.  $4001^8$

C. 4001

D.  $\frac{1}{3}$

E. NOTA

13. If  $\log_2(\log_3(\log_7(\log_{15} C))) = 13$ , then how many distinct positive prime numbers are factors of  $C$ ?

A. 1

B. 2

C. 3

D. 4

E. NOTA

14. Which of the following is equivalent to  $\sin(2x)$ ?

A.  $\frac{e^{2ix} - e^{-2ix}}{2i}$

B.  $\frac{e^{2ix} - e^{-2ix}}{-4}$

C.  $\frac{e^{ix} - e^{-ix}}{2i}$

D.  $\frac{e^{ix} - e^{-ix}}{-4}$

E. NOTA

15. Find the sum of the positive real solutions of  $x^2 = 2^x$ .

A. 6

B. 2

C. 4

D. 1

E. NOTA

16. Simplify:  $\log_{11} 625 \cdot \log_7 243 \cdot \log_5 14641 \cdot \log_3 16807$   
A. 1                      B. 400                      C. 80                      D. 100                      E. NOTA

17. To make Arjun stop talking during their road trip across Gainesville, Sidhika must solve this:

$\lim_{x \rightarrow 3} \frac{\sqrt{x} - \sqrt{3}}{x - 3}$ . What answer does she come up with, assuming she answers it correctly?  
A.  $\frac{-\sqrt{3}}{6}$                       B. 0                      C.  $\frac{\sqrt{3}}{6}$                       D. 1                      E. NOTA

18. Let  $f(x) = \sqrt{\frac{\cos^2(x) - \sin^2(x)}{1 - \tan^2(x)}}$ . Evaluate  $f\left(\frac{3\pi}{8}\right)$ .  
A.  $\frac{\sqrt{2-2\sqrt{2}}}{2}$                       B.  $-\frac{\sqrt{2-\sqrt{2}}}{2}$                       C.  $\frac{\sqrt{2+\sqrt{2}}}{2}$                       D.  $-\frac{\sqrt{2+\sqrt{2}}}{2}$                       E. NOTA

Questions 19 and 20 pertain to the following function:

$$f(x) = \sin^{-1}\left(\log_2 \frac{x^2}{2}\right)$$

19. What is the domain of  $f(x)$ ?  
A.  $[-2, -1] \cup (1, 2]$                       B.  $(-1, \infty)$                       C.  $(-\infty, \infty)$                       D.  $[-2, -1] \cup [1, 2]$                       E. NOTA

20. What is the range of  $f(x)$ ?  
A.  $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$                       B.  $[0, \pi]$                       C.  $[-\pi, \pi]$                       D.  $[2, \infty)$                       E. NOTA

21. If  $\log_2 a = \log_3 b = \log_6 c = \log_7(a + b + c)$ , find  $\log_{abc} 6$ .  
A. 2                      B. 1                      C.  $\frac{1}{4}$                       D.  $\frac{1}{2}$                       E. NOTA

22. What is the domain of the function  $f(x) = \frac{1}{\log(1-x)} + \sqrt{x+2}$ ?  
A.  $[-2, \infty)$                       B.  $[-2, 1)$                       C.  $[-2, 0) \cup (0, 1)$                       D.  $[-\infty, 0) \cup (0, 1]$                       E. NOTA

23. Find the product of the real values of  $x$  for which  $\log_3(x - 5) - \log_{27}(79x - 185) = 0$ .  
A. -2                      B. 15                      C. 60                      D. -60                      E. NOTA

24. Determine  $\tan\left(\csc^{-1}\frac{e^x}{2\ln x}\right)$ , where  $x > 1$ .
- A.  $\frac{\sqrt{e^{2x}+4(\ln x)^2}}{2\ln x}$     B.  $\frac{2\ln x}{\sqrt{e^{2x}-4(\ln x)^2}}$     C.  $\frac{2\ln x}{\sqrt{e^{2x}+4(\ln x)^2}}$     D.  $\frac{\sqrt{e^{2x}-4(\ln x)^2}}{2\ln x}$     E. NOTA
25. If the function  $f$  on the domain  $\left[\frac{1}{2}, \infty\right)$  is defined by  $f(x) = 2^{x(x-1)}$ , then what is  $f^{-1}(x)$ , the inverse of the function?
- A.  $\frac{1}{2}(1 + \sqrt{1 + 4\log_2 x})$     B.  $\frac{1}{2}(1 - \sqrt{1 + 4\log_2 x})$     C.  $\sqrt{1 + 4\log_2 x}$     D.  $\sqrt{1 - 4\log_2 x}$     E. NOTA
26. The function  $\log(x + \sqrt{x^2 + 1})$  is
- A. Neither even nor odd    B. Even    C. Periodic    D. Odd    E. NOTA
27. What is the  $y$ -intercept of the line tangent to  $f(x) = e^{\cos x}$  at  $x = \frac{\pi}{2}$ ? The slope of the tangent line can be found by plugging an  $x$ -value into  $f'(x) = -\sin(x)e^{\cos x}$ .
- A.  $\frac{\pi}{2}$     B. 1    C. -1    D. 0    E. NOTA
28. If  $x$  satisfies  $2x - \log x + \ln x = 0$ , then  $x \log_x 10$  can be expressed in the form  $a(1 + \ln b)$ . Compute  $\frac{a}{b}$ .
- A.  $\frac{1}{20}$     B.  $\frac{1}{5}$     C. 5    D. 20    E. NOTA
29. The formula for the Richter Scale is  $\log\frac{A}{A_0}$ , where  $A$  is the measured amplitude of the earthquake, and  $A_0$  is the amplitude of the smallest detectable wave (or standard wave). If the earthquake Shamraj is tracking follows the function  $150 \sin x + 360 \cos x$ , and has a standard wave amplitude of 1.95, what estimate will Shamraj calculate for the magnitude of this earthquake? (Note: Shamraj uses the approximation  $\log 2 \approx 0.301$ .)
- A. 2.5    B. 1.301    C. 2.301    D. 0.301    E. NOTA
30. The domain of the function  $f(x) = \sin\left(\ln\frac{\sqrt{4-x^2}}{1-x}\right)$  is:
- A. (1, 2)    B.  $(-\infty, 1) \cup (1, \infty)$     C.  $[-2, 1) \cup (1, 2]$     D.  $(-2, 1)$     E. NOTA