For all questions below, the answer E. NOTA means "None of these answers".

1. Write the system $\begin{cases} 2x + 3y = 8 \\ x - 2y = -3 \end{cases}$ as a matrix equation:

A. $\begin{bmatrix} x \\ y \end{bmatrix}\begin{bmatrix} 8 \\ -3 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 1 & -2 \end{bmatrix}$ B. $\begin{bmatrix} 2 & 3 \\ 1 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ -3 \end{bmatrix}$ C. $\begin{bmatrix} x \\ y \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 1 & -2 \end{bmatrix} = \begin{bmatrix} 8 \\ -3 \end{bmatrix}$

D. $\begin{bmatrix} 2 & 1 \\ 3 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ -3 \end{bmatrix}$ E. NOTA

2. Determine the product $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} -1 & 2 \\ 3 & -4 \end{bmatrix}$.

A. $\begin{bmatrix} 5 & -6 \\ 9 & -10 \end{bmatrix}$ B. $\begin{bmatrix} -1 & 4 \\ 9 & -16 \end{bmatrix}$ C. $\begin{bmatrix} 5 & 6 \\ -9 & -10 \end{bmatrix}$

D. $\begin{bmatrix} 5 & 6 \\ 9 & 10 \end{bmatrix}$ E. NOTA

3. What is $\begin{bmatrix} -4 & 5 \\ 2 & -2 \end{bmatrix}^{-1}$?

A. $\begin{bmatrix} 4 & -5 \\ -2 & 2 \end{bmatrix}$ B. $\begin{bmatrix} -\frac{1}{4} & \frac{1}{5} \\ \frac{1}{2} & -\frac{1}{2} \end{bmatrix}$ C. $\begin{bmatrix} 1 & \frac{5}{2} \\ 1 & 2 \end{bmatrix}$

D. $\begin{bmatrix} -\frac{1}{4} & 0 \\ 0 & -\frac{1}{2} \end{bmatrix}$ E. NOTA

4. The determinant of an n x n matrix A is $\det(A) = k$ for some real value k. Consider the matrix $B = kAA^TA^{-1}$. Which of the following equals $\det(B)$ in terms of k?

A. k^4 B. k^n C. k^{n+1} D. k^{n+2} E. NOTA

5. Which matrix is <u>not</u> singular?

A. $\begin{bmatrix} 1 & 2 & 3 \\ -1 & -2 & -3 \\ 4 & 5 & 2 \end{bmatrix}$ B. $\begin{bmatrix} 2 & 0 & -4 \\ -3 & 2 & 6 \\ 1 & 0 & -2 \end{bmatrix}$ C. $\begin{bmatrix} 2 & 1 & 3 \\ 1 & 2 & -3 \\ 3 & 3 & 0 \end{bmatrix}$

D. $\begin{bmatrix} -2 & 3 & 1 \\ 0 & 0 & 0 \\ 1 & 2 & 8 \end{bmatrix}$ E. NOTA

6. Find the value of *k* so that the system is inconsistent: $\begin{cases} 2x + ky = 4 \\ -3x - 2y = -6 \end{cases}$

A. $k = 3$ B. $k = -3$ C. $k = \frac{4}{3}$ D. $k = -\frac{4}{3}$ E. NOTA

7. The matrix $A = \begin{bmatrix} 2 & -5 \\ 3 & 1 \end{bmatrix}$ satisfies the quadratic equation $A^2 + \alpha A + \beta I = 0$, where α and β are integers and *I* is the 2x2 identify matrix. Determine the value of $\alpha + \beta$.

A. -14 B. 11 C. 14 D. 20 E. NOTA

8. Let $A = \begin{bmatrix} \sqrt{3} & -1 \\ 1 & \sqrt{3} \end{bmatrix}$. What is A^7?

A. $\begin{bmatrix} 64\sqrt{3} & -64 \\ 64 & 64\sqrt{3} \end{bmatrix}$ B. $\begin{bmatrix} -64\sqrt{3} & 64 \\ -64 & -64\sqrt{3} \end{bmatrix}$ C. $\begin{bmatrix} 128\sqrt{3} & -128 \\ 128 & 128\sqrt{3} \end{bmatrix}$

D. $\begin{bmatrix} -128\sqrt{3} & 128 \\ -128 & -128\sqrt{3} \end{bmatrix}$ E. NOTA

9. Which matrix is written in reduced row echelon form?

A. $\begin{bmatrix} 1 & 0 & 2 & 3 \\ 0 & 0 & 0 & 0 \\ -2 & 1 & -1 & 3 \end{bmatrix}$ B. $\begin{bmatrix} 1 & 2 & 1 & -2 \\ 0 & 1 & 3 & 9 \\ 0 & 0 & 1 & 2 \end{bmatrix}$ C. $\begin{bmatrix} 5 & 2 & -1 & 3 \\ 0 & 1 & 5 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

D. $\begin{bmatrix} 1 & 0 & 2 & 4 \\ 0 & 1 & -1 & -8 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ E. NOTA

10. Find the area of the triangle with vertices at $(-2, 1, 3)$, $(1, 5, 4)$, and $(5, 2, 5)$.

A. $5\sqrt{3}$ B. $\frac{15\sqrt{3}}{2}$ C. 44 D. 88 E. NOTA

11. Find the sum of the solutions to the equation: $\begin{vmatrix} -3 & x+3 \\ 2-x & 2 \end{vmatrix} = 8$

A. -5 B. -1 C. 1 D. 4 E. NOTA

12. In $M = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, each entry *a, b, c, d* is independently and randomly assigned a value of 0 or 1 with equal probability. What is the probability that *M* is invertible?

A. $\frac{1}{8}$ B. $\frac{1}{4}$ C. $\frac{3}{8}$ D. $\frac{1}{2}$ E. NOTA

13. Solve for matrix *A*: $2A + \begin{bmatrix} -1 & 0 \\ -1 & 3 \\ 2 & -4 \end{bmatrix} = \begin{bmatrix} 3 & 4 \\ -3 & -1 \\ 8 & -4 \end{bmatrix}$

A. $\begin{bmatrix} 1 & 2 \\ -2 & 2 \\ 5 & -4 \end{bmatrix}$ B. $\begin{bmatrix} 2 & 2 \\ -1 & -2 \\ 3 & -4 \end{bmatrix}$ C. $\begin{bmatrix} 4 & 4 \\ -2 & -4 \\ 6 & 0 \end{bmatrix}$ D. $\begin{bmatrix} 2 & 2 \\ -1 & -2 \\ 3 & 0 \end{bmatrix}$ E. NOTA

14. Let *S* be the set of all 2×2 matrices with real entries. Consider the following properties:
I. $AB = BA$ II. $(AB)C = A(BC)$ III. $A(B + C) = AB + AC$.
Which of these properties are true for all matrices *A, B, C* in *S*?

A. I only B. II only C. II and III only D. I and III only E. NOTA

15. Using Cramer's Rule, $x = \frac{\begin{vmatrix} 2 & 3 \\ -1 & 5 \end{vmatrix}}{\begin{vmatrix} 4 & 3 \\ 1 & 5 \end{vmatrix}}$ is the solution for *x* in a 2×2 system involving *x* and *y*. What is this 2x2 system of equations?

A. $\begin{cases} 2x + 3y = 3 \\ -x + 5y = 5 \end{cases}$ B. $\begin{cases} 2x + 3y = 4 \\ -x + 5y = 1 \end{cases}$ C. $\begin{cases} 2x - y = 4 \\ 3x + 5y = 1 \end{cases}$ D. $\begin{cases} 4x + 3y = 2 \\ x + 5y = -1 \end{cases}$ E. NOTA

16. Which of the following 3-D vectors has a magnitude that is a positive integer?

A. $\begin{pmatrix} -2 \\ 2 \\ -4 \end{pmatrix}$ B. $\begin{pmatrix} 2 \\ -4 \\ 6 \end{pmatrix}$ C. $\begin{pmatrix} 1 \\ 4 \\ -8 \end{pmatrix}$ D. $\begin{pmatrix} -6 \\ 7 \\ 7 \end{pmatrix}$ E. NOTA

17. Which of the following is a unit vector in the *opposite direction* of $\vec{v} = \begin{pmatrix} -1 \\ 2 \\ -2 \end{pmatrix}$?

A. $\begin{pmatrix} \frac{1}{3} \\ -\frac{2}{3} \\ \frac{2}{3} \end{pmatrix}$ B. $\begin{pmatrix} \frac{1}{9} \\ -\frac{2}{9} \\ \frac{2}{9} \end{pmatrix}$ C. $\begin{pmatrix} -\frac{1}{3} \\ \frac{2}{3} \\ -\frac{2}{3} \end{pmatrix}$ D. $\begin{pmatrix} \frac{1}{5} \\ -\frac{2}{5} \\ \frac{2}{5} \end{pmatrix}$ E. NOTA

18. Find the *sum* of all values of x that make $\begin{bmatrix} -1 & x & -3 \\ x & 0 & 0 \\ 3 & -2 & x+1 \end{bmatrix}$ not invertible.

A. 1 B. 0 C. -1 D. -6 E. NOTA

19. If $\vec{u} =< 2, -3 >$ is perpendicular to both $\vec{v} =< x, -2 >$ and $\vec{w} =< -1, 2y >$, find the value of xy.

A. -2 B. -1 C. 0 D. 1 E. NOTA

20. Which matrix below is idempotent?

A. $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ B. $\begin{bmatrix} 1 & 0 & 1 \\ 2 & 0 & 0 \\ -1 & 1 & 1 \end{bmatrix}$ C. $\begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$ D. $\begin{bmatrix} 2 & 2 & -1 \\ -1 & 3 & 4 \\ -1 & 2 & 3 \end{bmatrix}$ E. NOTA

21. Which matrix below is nilpotent?

A. $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ B. $\begin{bmatrix} 5 & -3 & 2 \\ 15 & -9 & 6 \\ 10 & -6 & 4 \end{bmatrix}$ C. $\begin{bmatrix} 1 & 0 & 1 \\ 2 & 0 & 0 \\ -1 & 1 & 1 \end{bmatrix}$ D. $\begin{bmatrix} 5 & 1 & 2 \\ 5 & 1 & -2 \\ 5 & 1 & 0 \end{bmatrix}$ E. NOTA

22. Find the angle between the vectors $\vec{u} =< 3, -1 >$ and $\vec{v} =< 2,1 >$.

A. $30°$ B. $40°$ C. $45°$ D. $60°$ E. NOTA

23. Let $\vec{u} =< -1,4 >, \vec{v} =< 3,2 >, \vec{w} =< -2,1 >$. If $a\vec{u} + b\vec{v} + c\vec{w} = \vec{0}$ for some non-zero a, b, c, compute the distance from the origin to the line $ax + by = c$.

A 0 B. 1 C. $\sqrt{2}$ D. 2 E. NOTA

24. If the trace of a 4×4 matrix A is 3, what is the trace of $5A$?

A. 3 B. 12 C. 15 D. 240 E. NOTA

25. Given $|\vec{v} + \vec{w}| = m$ and $|\vec{v} - \vec{w}| = n$, find the dot product, $\vec{v} \cdot \vec{w}$, in terms of m and n.

A. $\frac{1}{4}(m^2 - n^2)$ B. $\frac{1}{4}(m^2 + n^2)$ C. $\frac{1}{4}(m - n)$ D. $\frac{1}{4}(m - n)^2$ E. NOTA

26. Which vector is linearly independent to $\begin{bmatrix} -2 \\ 4 \end{bmatrix}$?

A. $\begin{bmatrix} 1 \\ -2 \end{bmatrix}$ B. $\begin{bmatrix} -1 \\ 2 \end{bmatrix}$ C. $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ D. $\begin{bmatrix} 2 \\ -4 \end{bmatrix}$ E. NOTA

27. If $A = \begin{bmatrix} -2 & 3 & 7 \\ x & 5 & z \\ y & -2 & -1 \end{bmatrix}$ is symmetric, find the sum $x + y + z$.

A. 5 B. 6 C. 8 D. 10 E. NOTA

28. Find the product xyz in the system $\begin{cases} x + y = 8 \\ x + z = 11. \\ y + z = 13 \end{cases}$

A. 88 B. 104 C. 143 D. 150 E. NOTA

29. Which of the following must be true for any two $n \times n$ matrices A and B with real entries?

A. $(A + B)^2 = A^2 + 2AB + B^2$ B. $(A + B)^{-1} = A^{-1} + B^{-1}$
C. $tr(AB) = tr(A)tr(B)$ D. $(AB)^{-1} = A^{-1}B^{-1}$
E. NOTA

30. Suppose $\begin{bmatrix} 2 & -4 \\ -1 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = c\begin{bmatrix} x \\ y \end{bmatrix}$ and $\begin{bmatrix} x \\ y \end{bmatrix}$ is not the zero vector and c is some real number. Find the product of all possible values of c.

A. -6 B. -2 C. -1 D. 1 E. NOTA