Alpha nationals Bowl solutions 2019

Q1- $2x^2 = 288 \rightarrow x^2 = 144 \rightarrow x = \pm 12 \rightarrow (12,5),(12,-5),(-12,5),(-12,-5)$

A 24 B. -10

$$\frac{15}{x} - \frac{6}{y} = 15$$
$$\frac{-8}{x} + \frac{6}{y} = 6 \rightarrow \frac{7}{x} = 21 \rightarrow x = \frac{1}{3} \rightarrow y = \frac{1}{5} \rightarrow \frac{1}{15}$$

$$(24-10)15 = 210$$

Q2

R-Expansion begins with $(9x)^{1/2} + (1/2)(9x)^{-1/2}(-y) - (1/8)(9x)^{-3/2}(-y)^2$. The third term therefore has a coefficient of -1/216.

M $2^6 = 64$

P Go to the 6th row of Pascal's triangle and then add up the last 4 numbers 20+15+6+1=42

RMP = $\frac{64 \bullet 42}{-216} = \frac{-112}{9}$ $\frac{-112}{9} = -12\frac{4}{9}$

Q3

Z. $\frac{x+1}{x-1}\left[x^2 - 1 + \frac{x-1}{x+1}\right] = (x+1)^2 + 1 \rightarrow x^2 + 2x + 2$

L.
$$\frac{(x+2)^3(x^2+4x+3)(x^2-6x+8)(4)}{(x^2+x-2-x^2+x+6)(x^2+x-2+x^2-x-6)(x+1)(x+2)(x+3)}$$
$$\frac{(x+2)^3(x+1)(x+3)(x-4)(x-2)4}{2(x+2)2(x-2)(x+2)(x+1)(x+2)(x+3)} \rightarrow x-4$$

$$\frac{\left(x^2-4y^2+4y-1\right)\left(4y^2-x^2\right)}{\left(x^2-x-4y^2+2y\right)\left(x^2+2y+x-4y^2\right)}=$$

U. $$\frac{\left[x^2-\left(4y^2-4y+1\right)\right]\left(2y+x\right)\left(2y-x\right)}{\left(x^2-4y^2-x+2y\right)\left(x^2-4y^2+x+2y\right)}=$$

$$\frac{-1\left(x-2y+1\right)\left(x+2y-1\right)\left(x+2y\right)\left(x-2y\right)}{\left(x-2y\right)\left(x+2y-1\right)\left(x+2y\right)\left(x-2y+1\right)}=-1$$

$x^2+2x+2+x-4-1=x^2+3x-3$ -3-3= -6

Q4

J $$\lim_{k\to\infty}\frac{1+4+9+25+...+k^2}{\left(k-1\right)^3}=\frac{k\left(k+1\right)\left(2k+1\right)}{6\left(k-1\right)^3}=\frac{1}{3}$$

W $$\lim_{x\to 0}\frac{\frac{1}{x+4}-\frac{1}{4}}{x}=\frac{4-\left(x+4\right)}{4x\left(x+4\right)}=\frac{-1}{4\left(x+4\right)}=\frac{-1}{16}$$

I $$\lim_{x\to 2^+}\frac{x-2}{\sqrt{x^2-4}}=\frac{\left(x-2\right)\sqrt{x^2-4}}{x^2-4}=\frac{\sqrt{x^2-4}}{x+2}=0$$

G $$\lim_{x\to 7^-}\frac{\left|8x-56\right|}{7-x}=\frac{8\left|x-7\right|}{7-x}=\frac{8\left(7-x\right)}{7-x}=8$$

$$8\div\frac{-1}{48}+0=-384$$

Q5

S $$\frac{c}{a}=\frac{5}{4}\to 19--1=20=2c\to c=10\to a=8$$

$$b^2=100-64=36\to b=6\to 2b=12$$

L $3\left(x^2+2x+1\right)-4\left(y^2-2y+1\right)=-L+3-4\rightarrow L=-1$

H
$$x^2-6x+9=12y+51+9$$
$$\left(x-3\right)^2=12\left(y+5\right)\rightarrow 4p=12$$

12-1+12=23

Q6

Z $31+31\sqrt{2}=\dfrac{a_1\left(\sqrt{2}^{10}-1\right)}{\sqrt{2}-1}\rightarrow 31=a_1\left(32-1\right)\rightarrow a_1=1\rightarrow a_3=2$

L

1	6	25	70	153	286
5	19	45	83	133	
14	26	38	50		
12	12	12			286

U
$$\frac{-1}{\left(n-2\right)\left(n-3\right)}=\frac{A}{n-2}+\frac{B}{n-3}\rightarrow -1=A\left(n-3\right)+B\left(n-2\right)\rightarrow A=1,B=-1$$
$$\frac{1}{n-2}-\frac{1}{n-3}=\left(\frac{1}{2}-1\right)+\left(\frac{1}{3}-\frac{1}{2}\right)+\left(\frac{1}{4}-\frac{1}{3}\right)+...=-1$$

2+286-1=287

Q7

W $\dfrac{3}{4}\bullet\dfrac{8!}{2!\bullet 2!}=7560$

J
$$2019\div 7=288\rightarrow 288\div 7=41\rightarrow 41\div 7=5$$
$$288+41+5=334$$

$$\frac{2019}{7} > \frac{x+y}{x} > \frac{2019}{8}$$

F $$288\frac{3}{7} > 1+\frac{y}{x} > 252\frac{3}{8} \rightarrow 287\frac{3}{7} > \frac{y}{x} > 251\frac{3}{8}$$

$$[252, 287] \rightarrow 36$$

7560+334+36=7930

Q8

w $x^5 = 1 \rightarrow x^5 - 1 = 0$ so sum =0 and therefore the rest equal -1

$$5-12i = (a+bi)^2 = a^2 - b^2 + 2abi$$

I $$a^2 - b^2 = 5 \rightarrow 2ab = -12 \rightarrow ab = -6$$

$$3-2i, -3+2i \rightarrow (3)(-2)(-3)(2) = 36$$

$$(x-2+i)(x-2-i) = x^2 - 4x + 5$$

J $$x^4 - 2x^3 - 6x^2 + 22x - 15 = (x^2 - 4x + 5)(x^2 + 2x - 3)$$ sum of reals is 1-3=-2

(-1)(36)(-2)= 72

Q9

Z $$\log_{18} 4 + \log_{18} 6 + 5\log_{18} 3 = \log_{18}(24 \bullet 3^5) = 3$$

$$(\log x)^2 - 2\log x < 0 \rightarrow (\log x)(\log x - 2) < 0$$

L $$1 < x < 100 \rightarrow [2, 99] \rightarrow 98$$

$$\log_4 (x+1)^2 + \log_8 (x-3)^3 - \log_{16} 12^4 = 0$$

$$\log_2 (x+1) + \log_2 (x-3) - \log_2 12 = 0$$

U $$\log_2\left(\frac{(x+1)(x-3)}{12}\right) = 0 \rightarrow (x+1)(x-3) = 12$$

$$(x-5)(x+3) = 0 \rightarrow 5, -3 \rightarrow 5$$

3+98+5=106

Q10

$$M=\frac{1}{12-18}\begin{pmatrix}3 & -9\\ -2 & 4\end{pmatrix}\begin{pmatrix}-4 & 5\\ -2 & 7\end{pmatrix}=\frac{-1}{6}\begin{pmatrix}6 & -48\\ 0 & 18\end{pmatrix}\rightarrow 4$$

A= $\begin{vmatrix}1 & 3 & -2 & -1\\ 0 & 2 & 4 & 1\\ 0 & -5 & 0 & 3\\ -1 & -3 & -1 & 1\end{vmatrix}=\begin{vmatrix}0 & 0 & -3 & 0\\ 0 & 2 & 4 & 1\\ 0 & -5 & 0 & 3\\ -1 & -3 & -1 & 1\end{vmatrix}=\begin{vmatrix}0 & -3 & 0\\ 2 & 4 & 1\\ -5 & 0 & 3\end{vmatrix}=3\begin{vmatrix}2 & 1\\ -5 & 3\end{vmatrix}=3(6+5)=33$

O $\begin{bmatrix}x & y\\ y & x\end{bmatrix}\begin{bmatrix}x & y\\ y & x\end{bmatrix}=\begin{bmatrix}5 & 4\\ 4 & 5\end{bmatrix}\rightarrow x^2+y^2=5\rightarrow 2xy=4$

$(1,2)(2,1)(-1,-2)(-2,-1)\rightarrow 4$

4+4+33= 41

Q11

W $\quad \sin x+1-2\sin^2 x\rightarrow\frac{-b}{2a}=\frac{1}{4}\rightarrow\frac{1}{4}+1-\frac{1}{8}=\frac{9}{8}$

J $\quad \frac{J}{377}=\left(\frac{21}{29}\right)\left(\frac{12}{13}\right)-\left(\frac{5}{13}\right)\left(\frac{20}{29}\right)=\frac{152}{377}\rightarrow 152$

F $\quad \frac{\tan x+\tan y}{1-\tan x\tan y}=\frac{\frac{5}{12}-\frac{3}{4}}{1-\left(\frac{5}{12}\right)\left(\frac{-3}{4}\right)}=\frac{20-36}{48+15}=\frac{-16}{63}\rightarrow -16$

$\frac{9}{8}\bullet -16+152=134$

Q12

M

	Lu-it is yes	Lu-it is no	total
Test + yes	.049	.095	.144
Test + no	.001	.855	.856
total	.05	.95	1.

$$\frac{.049}{.144} = \frac{49}{144}$$

A

	1	2	3	4	5	6	7	8
1	x				x			
2		x		x				
3			x					
4		x		x				
5	x				x			
6						x		

$$\frac{10}{48} = \frac{5}{24}$$

O The child options are: BB, GB, BG, and GG. The condition one is a boy leaves: BB, GB, and BG. Only 1 of these 3 satisfies the problem. so $\frac{1}{3}$

$$\frac{49}{144} + \frac{5}{24} \div \frac{1}{3} = \frac{49+30}{144} \bullet 3 = \frac{79}{48} = 1\frac{31}{48}$$

Q13

J

$$1 + \tan^2 x - 2\tan x - 4 = 0$$
$$\tan^2 x - 2\tan x - 3 = 0$$
$$(\tan x - 3)(\tan x + 1) = 0 \to 4$$

P

$$\cot x\left(\cos^2 x - 1\right) = 0$$
$$\cot x\left(-\sin^2 x\right) = 0$$ watch for extraneous solution with sinx = 0
$$0 + 2\pi = 2\pi$$

F $\text{Sin}^{-1}\left(\frac{-\sqrt{3}}{2}\right)=\frac{-\pi}{3}$

R -1136+4(360)=314

$$\frac{4(2\pi)}{\frac{-\pi}{3}}+314=-24+314=290$$

Q14

$$\frac{12}{b-2}+\frac{12}{b+2}=\frac{25}{b}\rightarrow b=10$$

$$\frac{260}{r+13}+1=\frac{260}{r}\rightarrow r=52$$

$$AB=(10)(52)=520$$

Answers:

1. 210

2. $\frac{-112}{9} = -12\frac{4}{9}$

3. -6

4. -384

5. 23

6. 287

7. 7930

8. 72

9. 106

10. 41

11. 134

12. $\frac{79}{48} = 1\frac{31}{48}$

13. 290

14. 520