Throughout this test, the notation $\{a_n\}_{n\geq 0}$ refers to a sequence of numbers $a_0, a_1, a_2, \ldots$ A reward for reading the instructions: the more distinct positive factors N has, the more difficult the N^{th} question on the test is. For example, 1 has a single factor, so the first question is the easiest. On the other hand, 24 and 30 have eight factors, so questions 24 and 30 are the hardest.

1. An arithmetic sequence has third term 30 and common difference 7. What is the first term?
   A. 9 B. 16 C. 23 D. 44 E. NOTA

2. Evaluate the expression:
   $$107 + 64 + 84 + 1923 - 67 - 24 - 44 - 1883$$
   A. 160 B. 170 C. 180 D. 190 E. NOTA

3. Three distinct positive integers are in geometric progression. Find the minimum possible value of their sum.
   A. 3 B. 6 C. 7 D. 19 E. NOTA

4. Let $\{a_n\}_{n\geq 0}$, $\{b_n\}_{n\geq 0}$, and $\{c_n\}_{n\geq 0}$ be arithmetic progressions. If $a_0 + b_0 + c_0 = 3$ and $a_1 + b_1 + c_1 = 8$, then what is $a_{2019} + b_{2019} + c_{2019}$?
   A. 10093 B. 10095 C. 10098 D. 10103 E. NOTA

5. Let S be the set of integral powers of 3. Let T be the set of integers greater than 1. Find the sum of the 6 smallest elements in $S \cap T$.
   A. 1090 B. 1091 C. 1092 D. 1093 E. NOTA

6. Consider an arithmetic sequence $\{a_n\}_{n\geq 1}$ such that $a_1 = 3$ and $a_2 = 7$. Find the value of $a_{a_1} + a_{a_2} + a_{a_3} + a_{a_4} + a_{a_5}$.
   A. 55 B. 75 C. 195 D. 215 E. NOTA

7. An arithmetic sequence has first term 2 and second term 6. Find the sum of the first 30 terms.

A. 1800 B. 2700 C. 2800 D. 3600 E. NOTA

8. Call a set *special* if it consists of exactly 33 distinct positive integers, all less than or equal to 100. Let m be the smallest number in a given special set. What is the largest possible value of m?

A. 33 B. 34 C. 67 D. 68 E. NOTA

9. The sum of the first n terms of a sequence is equal to $n^3 + 3n + 1$. Find the 10^{th} term of the sequence.

A. 274 B. 334 C. 1031 D. 1365 E. NOTA

10. Let a *barbershop quartet* be four prime numbers in arithmetic progression. Which of the following primes is not in any barbershop quartet?

A. 3 B. 5 C. 7 D. 11 E. NOTA

11. Ellen's teacher gave her an arithmetic sequence with first term 1 and common difference d. However, Ellen accidentally wrote down a sequence with first term $2d$ and common difference 1. If Ellen still managed to correctly find the fifth term of the sequence, what is d?

A. 0.5 B. 1 C. 1.5 D. 2 E. NOTA

12. Find the coefficient of x^6 in the expansion of

$$(1 + 2x + 4x^2 + 8x^3 + \dots)(1 + 3x + 9x^2 + 27x^3 + \dots)$$

A. 665 B. 1296 C. 1458 D. 2059 E. NOTA

13. Find the sum of the 200 smallest odd natural numbers.
A. 39600 B. 40000 C. 40400 D. 44000 E. NOTA

14. Real numbers a, b, and c, with $a < b < c$, are in arithmetic progression, while the numbers a, b, and $3c$ are in geometric progression, in that order. Find the value of $\frac{b}{a}$.
A. $1+\sqrt{2}$ B. 3 C. $3+\sqrt{6}$ D. $2+\sqrt{5}$ E. NOTA

15. The integers $1, 2, \ldots, n$ are written in order on a slip of paper. The slip is then cut into four pieces so that each piece consists of some nonempty set of consecutive integers, and each of the n integers is on exactly one of the four pieces. The averages of the numbers on the pieces are 6.5, 24, 98, and 232. Compute n.
A. 303 B. 330 C. 361 D. 463 E. NOTA

16. Let r_n be the remainder when $(n+1)^2$ is divided by n^2. Find the remainder when the sum $r_2 + r_3 + r_4 + \ldots + r_{2019}$ is divided by 1000.
A. 353 B. 357 C. 392 D. 396 E. NOTA

17. Find the eleventh term of a geometric progression with seventh term 28 and third term 7.
A. 56 B. $56\sqrt{2}$ C. $112\sqrt{2}$ D. 112 E. NOTA

18. The numbers $\sin\alpha$, $\cos\alpha$, and $\sin 2\alpha$ are in geometric progression (in that order). Given that $\cos\alpha$ is nonzero, find its value.
A. $\frac{-1-\sqrt{17}}{4}$ B. $\frac{-1+\sqrt{17}}{4}$ C. $\frac{-1-\sqrt{2}}{4}$ D. $\frac{-1+\sqrt{2}}{4}$ E. NOTA

19. Evaluate $-1-2+3+4-5-6+7+8-9-10+\ldots+99+100$.
A. 80 B. 100 C. 125 D. 200 E. NOTA

20. A sequence $\{a_n\}_{n\geq 1}$ is *Fibonacci-like* if it satisfies $a_n = a_{n-1} + a_{n-2}$ for $n > 2$. If the sum of the first ten terms of a Fibonacci-like sequence is 517, which of the following is a possible value of the seventh term?
A. 44 B. 45 C. 46 D. 47 E. NOTA

21. Which of the following is equal to $\sin 18^\circ + \sin 90° + \sin 162\,° + \sin 234\,°$?
A. $\sin 18°$ B. $\sin 36°$ C. $\sin 54°$ D. $\sin 72°$ E. NOTA

22. In the sequence $Z, H, A, O, L, I, 7$, every three consecutive terms add up to 12. Compute $Z + H + O + I$.
A. 13 B. 16 C. 19 D. 24 E. NOTA

23. An arithmetic sequence has first term 1 and common difference 2. A geometric sequence has first term 1 and common ratio 2. Find the positive difference between the fifth terms of the two sequences.
A. 7 B. 8 C. 21 D. 22 E. NOTA

24. Let the sequence $\{a_n\}_{n\geq 0}$ be defined by $a_0 = \frac{1}{2}$ and $a_n = 1 + (a_{n-1} - 1)^2$ for $n \geq 1$. Find the infinite product:

$$\prod_{k=0}^{\infty} a_k = a_0 a_1 a_2 \ldots$$

A. 1 B. $\frac{5}{6}$ C. $\frac{3}{4}$ D. $\frac{2}{3}$ E. NOTA

25. The first five terms of an arithmetic sequence sum to 35. What's the third term?
A. 3 B. 5 C. 7 D. 8 E. NOTA

26. In a geometric series, the average of the first three terms is equal to the average of the next six terms (that is, the fourth through ninth terms). Find the sum of all possible real values of the common ratio.

A. $1+\sqrt[3]{2}$ B. $1-\sqrt[3]{2}$ C. $-1+\sqrt[3]{2}$ D. $-1-\sqrt[3]{2}$ E. NOTA

27. Let $I = 2019^{2019}$ and consider the sequence $I, \ln(I), \ln(\ln(I)), \ldots$ How many terms of this sequence are well-defined?

A. 4 B. 5 C. 6 D. 7 E. NOTA

28. Let $a_1, a_2, \ldots, a_{10}$ be positive integers that sum to 40. Find the maximum possible value of $a_1a_2 + a_2a_3 + \ldots + a_9a_{10}$.

A. 144 B. 173 C. 279 D. 294 E. NOTA

29. A geometric series has first term 4 and common ratio 8. Find the product of the first 10 terms.

A. 2^{155} B. 2^{156} C. 2^{158} D. 2^{160} E. NOTA

30. Let $z(x) = x^2 + c$, where c is a real number. Consider the sequence

$$0, z(0), z(z(0)), z\left(z(z(0))\right), \ldots$$

Find the largest value of c such that this sequence is bounded. (A sequence is *bounded* if there exists a real number K such that no member of the sequence has absolute value bigger than K.)

A. $\frac{1}{4}$ B. $\frac{1}{\pi}$ C. $\frac{1}{e}$ D. $\frac{1}{2}$ E. NOTA