1. Determine the area enclosed by the polar graph $r = 5 \cos \theta - 8 \sin \theta$.

    A. $\frac{13\pi}{2}$ B. 13π C. $\frac{89\pi}{4}$ D. 89π E. NOTA

2. Find the number of ordered pairs (x, y) satisfying $\sin\left(\frac{\pi x^2}{3}\right) = 1$ and $|x| + |y| = 2$.

    A. 1   B. 2   C. 4   D. 0   E. NOTA

3. If θ is the angle between the two vectors $\mathbf{a} = <2, 4, 5>$ and $\mathbf{b} = <3, 2, 7>$, what is the value of $\cos \theta$?

    A. $\frac{61\sqrt{310}}{930}$ B. $1/3$ C. $\frac{53}{930}$ D. $\frac{49\sqrt{310}}{930}$ E. NOTA

4. Simplify the following trigonometric expression: $\frac{\cos 25° \cos 65° \cos 50° \cos 100° \cos 200°}{\sin 40°}$

    A. ¼   B. $1/16$ C. $\cos 50°$ D. $\cos 100°$ E. NOTA

5. If $\tan x + \tan y = 6$ and $\cot x + \cot y = 7$, find the value of $\tan(x + y)$.
    A. 20   B. $7/6$ C. 42   D. $6/7$ E. NOTA

6. Find the range of the following function:
    $$f(x) = 3 \cos^2 x + \cos^4 x + 3 \sin^2 x + \sin^2 x \cos^2 x$$

    A. $[0,1]$ B. $[3,4]$ C. $(-\infty, \infty)$ D. $[3, \infty)$ E. NOTA

7. If a triangle with sides $p, q,$ and r has an incircle of radius $\frac{q+r-p}{2}$, what is $m\angle P$?

    A. $\pi/2$ B. $\pi/4$ C. $\pi/6$ D. $\pi/3$ E. NOTA

8.   Find the number of solutions of the equation $x^3 + x^2 + 4x + 2 \sin x = 0$ in $[0, 2\pi]$.

   A. 0   B. 1   C. 3   D. 5   E. NOTA

9.   Find $\sin 105°$.

   A. $\frac{\sqrt{6}+\sqrt{2}}{4}$ B. $\frac{\sqrt{6}}{3}$ C. $\frac{1}{2}$ D. $\frac{\sqrt{6}-\sqrt{2}}{4}$ E. NOTA

10.   Find the value of $\cos\frac{\pi}{7} \cdot \cos\frac{2\pi}{7} \cdot \cos\frac{4\pi}{7}$.

   A. $-\frac{1}{16}$ B. $-\frac{1}{8}$ C. $-\frac{1}{4}$ D. $-\frac{1}{2}$ E. NOTA

11.   Simplify: $\tan\left(\arccos\left(\sin\left(-\frac{\pi}{6}\right)\right)\right)$

   A. $-\sqrt{3}$ B. $-\frac{\sqrt{3}}{3}$ C. $\frac{\sqrt{3}}{3}$ D. $\sqrt{3}$ E. NOTA

12.   Evaluate the sum: $\sum_{n=1}^{\infty} \sin^{-1}\left(\frac{\sqrt{n}-\sqrt{n-1}}{\sqrt{n(n+1)}}\right)$

   A. $3 - \sqrt{3}$ B. π C. $\pi/8$ D. $\pi/2$ E. NOTA

13.   Evaluate the sum: $\sum_{k=1}^{2019}\left(\text{cis}\left(\frac{\pi k}{2019}\right)\right)^{4038}$

   A. 0   B. 2019   C. 4038   D. 1   E. NOTA

14.   Evaluate the product: $\prod_{n=1}^{89}(\tan n° \cos 1° + \sin 1°)$

   A. 0   B. $\csc 1°$ C. 1   D. $\sec 1°$ E. NOTA

15. Which of the following functions is/are odd?

    I. $\sec x \tan x$
    II. $\cot x$
    III. $\cos^3 x$
    IV. $x^3 \sin^2 x \cos x$

A. I, II, III, IV   B. I, II   C. I, IV   D. II, III   E. NOTA

16. If $\cos x = \frac{4}{5}$ and $-\frac{\pi}{2} < x < 0$, find $\cos\left(x + \frac{5763\pi}{2}\right)$.

A. $\frac{4}{5}$ B. $\frac{3}{5}$ C. $-\frac{3}{5}$ D. $-\frac{4}{5}$ E. NOTA

17. Find $\dfrac{\cos(64°)\cos(4°) - \cos(86°)\cos(26°)}{\cos(71°)\cos(41°) - \cos(49°)\cos(19°)}$.

A. -1   B. 0   C. $\frac{\sqrt{3}}{2}$ D. $\frac{1}{2}$ E. NOTA

18. Find the sum of solutions to the equation $\sin^3 x + \cos^3 x = \frac{1}{2}(\sin x + \cos x)$, where $x \in [0, 2\pi)$.

A. 4π B. π C. 2π D. 5π E. NOTA

19. Find the exact value of

$$\cos 135° + \sin\frac{7\pi}{6} - \cot 300° - \sec\frac{11\pi}{6} + \csc 45° - \tan\frac{7\pi}{4}$$

A. $\frac{1}{2} - \frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{3}$ B. $\frac{1}{2} + \frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{3}$ C. $\frac{-\sqrt{2}}{2} - \frac{\sqrt{3}}{3}$ D. 1   E. NOTA

20. Find the minimum value of $\dfrac{9x^2 \sin^2 x + 4}{x \sin x}$ for $x \in (0, \pi)$.

A. $2\sqrt{3} - 3$ B. 1   C. 24   D. 12   E. NOTA

21. If $\cos x = -\frac{4}{5}$, $\sin x > 0$, $\sin y = -\frac{5}{13}$, $\tan y > 0$, compute $\sin(x + y)$.

A. $-\frac{56}{65}$ B. $-\frac{16}{65}$ C. $\frac{16}{65}$ D. $\frac{56}{65}$ E. NOTA

22. Compute $\cos\left(\frac{2\pi}{2019}\right) + \cos\left(\frac{4\pi}{2019}\right) + \cos\left(\frac{6\pi}{2019}\right) + \cos\left(\frac{8\pi}{2019}\right) + \cdots + \cos\left(\frac{2018\pi}{2019}\right)$.

A. -1 B. $-\frac{1}{2}$ C. $\frac{1}{2}$ D. 1 E. NOTA

23. How many petals are in the graph of $r = 7(\cos^2 9\theta - \sin^2 9\theta)$?

A. 9   B. 18   C. 36   D. 81   E. NOTA

24. Steve flies a superjet with supersonic speed and leaves an airport traveling south at 1200 mph. A helicopter leaves the same airport at the same time traveling 60 degrees west of north at 300 mph. The superjet has the fuel to fly for 4 hours at that speed. What is the maximum distance Steve can travel south (assuming he loses no time or speed when turning) so that the fuel remaining will allow him to catch up to the helicopter?

A. 1500   B. 1800   C. 2000   D. 2250   E. NOTA

25. Find the value of the series: $\sum_{n=1}^{\infty}\left(\tan^{-1}\sqrt{n} - \tan^{-1}\sqrt{n+1}\right)$

A. $\pi/2$ B. $\pi/4$ C. $-\pi/2$ D. $-\pi/4$ E. NOTA

26. Find the period of the function $f(x) = \cos(\cos x)$.

A. 2π B. $\pi/2$ C. π D. $\pi/4$ E. NOTA

27. Calculate $\lim_{x \to 0} \frac{\sin x}{\sin 2x}$.

A. 0   B. 1/2   C. DNE   D. 1   E. NOTA

28. How many values of x, with $2 < x < 19$, satisfy the equation $\cos^2 x + 2\sin^2 x = 1$?
    A. 0   B. 2   C. -1   D. ½   E. NOTA

29. Find the period of $2\sin(4\pi x) + 6\cos(5\pi x)$.

    A. $\pi/2$ B. $\pi/3$ C. 2   D. 4/5   E. NOTA

30. Simplify $\tan 3x$ in terms of $\tan x$.
    A. $\dfrac{\tan^3 x - 3\tan x}{1 - \tan^2 x}$ B. $\tan^3 x - 3\tan x$ C. $\dfrac{\tan^3 x - 3\tan x}{1 - 3\tan^2 x}$ D. $\dfrac{\tan^3 x - 3\tan x}{\tan^2 x - 1}$ E. NOTA