Answers:

1. $9x^2 - 22x - 10$
2. $-\frac{4}{289}$
3. $\frac{9}{5}$
4. $\frac{2}{3}$
5. $(-4,2)$ (must be in interval notation)
6. $(-1,54)$
7. $\ln 2$
8. $\frac{\pi}{2}$
9. does not exist
10. $4e^e + e^{e-1}$ (or equivalent)
11. 12π
12. $\sqrt{e}$
13. $\frac{5\pi}{32}$
14. 16π
15. $512\sqrt{3}$
16. 360
17. 1.25 (must be in decimal form)
18. $y = e^{x^2}$
19. $y = \frac{e^x}{x^2}$ (or equivalent)
20. 21
21. 3.68 (must be in decimal form)
22. $\sqrt{e}$
23. 1
24. $-\frac{1}{3}x^3$
25. $-\frac{1}{2}$

Solutions:

1. Using the Constant, Constant Multiple, Sum/Difference, and Power Rules, $f'(x)=9x^2-22x-10$.

2. $g'(x)=\dfrac{(2x^3+1)(2x)-(x^2-1)(6x^2)}{(2x^3+1)^2}=\dfrac{-2x^4+6x^2+2x}{(2x^3+1)^2}$, so $g'(2)=\dfrac{-4}{17^2}=-\dfrac{4}{289}$.

3. $\lim\limits_{x\to-2}\dfrac{2x^2-10x-28}{3x^2+2x-8}=\lim\limits_{x\to-2}\dfrac{(2x-14)\cancel{(x+2)}}{(3x-4)\cancel{(x+2)}}=\dfrac{2(-2)-14}{3(-2)-4}=\dfrac{9}{5}$

4. $\lim\limits_{x\to\infty}\dfrac{2x^2-10x-28}{3x^2+2x-8}=\lim\limits_{x\to\infty}\dfrac{2-\frac{10}{x}-\frac{28}{x^2}}{3+\frac{2}{x}-\frac{8}{x^2}}=\dfrac{2-0-0}{3+0-0}=\dfrac{2}{3}$

5. $f'(x)=3x^2+6x-24=3(x+4)(x-2)$, which is negative for values of x in the interval $(-4,2)$, so this is the interval over which f is decreasing.

6. $f''(x)=6x+6$, which changes signs at $x=-1$, so this is the x-value for the point of inflection. $f(-1)=54$, so the point of inflection is at the point $(-1,54)$.

7. Using the substitution $u=x^2+1$, $\int_0^1\dfrac{2x}{x^2+1}dx=\int_1^2\dfrac{1}{u}du=\ln|u|\Big|_1^2=\ln2-\ln1=\ln2$.

8. $\int_0^1\dfrac{2}{x^2+1}dx=2\arctan x\Big|_0^1=2\arctan1-2\arctan0=2\cdot\dfrac{\pi}{4}-2\cdot0=\dfrac{\pi}{2}$

9. The limit is in indeterminate form of type $\dfrac{0}{0}$, so we will use L'Hôpital's Rule to evaluate:

$\lim\limits_{x\to0}\dfrac{\ln(1-x)-\sin x}{1-\cos^2 x}=\lim\limits_{x\to0}\dfrac{-\frac{1}{1-x}-\cos x}{2\sin x\cos x}$. At this point the numerator is headed toward -2 while the denominator is still headed toward 0, so we will examine the one-sided limits. As $x\to0$ from either side, the numerator will be negative as it is headed toward -2, but the denominator is negative on one side of 0 and positive on the other (in quadrant I, both $\sin x$ and $\cos x$ are positive, but in quadrant IV those quantities have opposite signs). Therefore, the two one-sided limits are ∞ and $-\infty$, meaning the limit does not exist.

10. Using logarithmic differentiation, $f'(x) = x^x(1+\ln x)$, and using the Product Rule on this function, $f''(x) = x^x\left((1+\ln x)^2 + \frac{1}{x}\right)$. Therefore, $f''(e) = e^e\left(4+\frac{1}{e}\right) = 4e^e + e^{e-1}$.

11. Based on the given information, $V = \pi r^2 h = \pi r^2 \cdot 2r = 2\pi r^3$. Differentiating both sides of this equation with respect to time, $\frac{dV}{dt} = 6\pi r^2 \frac{dr}{dt}$. Evaluating this expression using the given information, $\left.\frac{dV}{dt}\right|_{r=2,\frac{dr}{dt}=0.5} = 6\pi(2)^2(0.5) = 12\pi$ (all of the units are consistent).

12. Based on the given information, $\int_1^e \frac{1}{x}dx = 2\int_1^a \frac{1}{x}dx$. $\int_1^e \frac{1}{x}dx = \ln|x|\Big|_1^e = \ln e - \ln 1 = 1$, and $2\int_1^a \frac{1}{x}dx = 2\ln|x|\Big|_1^a = 2\ln a - 2\ln 1 = 2\ln a$. Therefore, $1 = 2\ln a \Rightarrow \ln a = \frac{1}{2} \Rightarrow a = \sqrt{e}$.

13. Since the power of sine is even, $\int_0^{\frac{\pi}{2}}(\sin^6 x)dx = \frac{5\cdot 3\cdot 1}{6\cdot 4\cdot 2}\cdot\frac{\pi}{2} = \frac{5\pi}{32}$.

14. Using the cylindrical shells method, the volume is $2\pi\int_0^2 x\left((9-x^2)-(1+x^2)\right)dx$ $= 2\pi\int_0^2(8x-2x^3)dx = 2\pi\left(4x^2 - \frac{1}{2}x^4\right)\Big|_0^2 = 2\pi(16-8-0+0) = 16\pi$.

15. Draw the base so that it is centered at the origin, and draw a cross-section vertically. Since the equation of the circle is $x^2+y^2=16$, the length of the cross-section from the upper edge to the x-axis is $\sqrt{16-x^2}$, making the entire length of the cross-section $2\sqrt{16-x^2}$. Since the area enclosed by a regular hexagon with side length s is $\frac{3\sqrt{3}}{2}s^2$, the area of the cross-section is $\frac{3\sqrt{3}}{2}\cdot 4(16-x^2) = 6\sqrt{3}(16-x^2)$. Therefore, the volume of the solid is $6\sqrt{3}\int_{-4}^4(16-x^2)dx = 12\sqrt{3}\int_0^4(16-x^2)dx = 12\sqrt{3}\left(16x - \frac{1}{3}x^3\right)\Big|_0^4$ $= 12\sqrt{3}\left(64 - \frac{64}{3} - 0 + 0\right) = 12\sqrt{3}\cdot\frac{128}{3} = 512\sqrt{3}$.

16. $P'(x)=2160+354x-x^2=(360-x)(6+x)$, so for the domain of the function, the only critical number is $x=360$. Further, P' changes from positive to negative at $x=360$, making this value a relative maximum. Since this is the only sign change for P' on the domain, this is also an absolute maximum.

17. Using Newton's Method, $x_1=x_0-\dfrac{f(x_0)}{f'(x_0)}$. Since $f'(x)=3x^2+1$, *f* is only increasing and therefore only has one real solution, and $x_1=1-\dfrac{1^3+1-3}{3\cdot 1^2+1}=\dfrac{5}{4}=1.25$.

18. Separating the variables, $\dfrac{dy}{y}=2xdx \Rightarrow \ln|y|=x^2+c \Rightarrow y=Ce^{x^2}$. Since $y(0)=1$, $y=e^{x^2}$.

19. Recognizing that the left-hand side of this equation is the result of differentiating a product makes this problem easier. $\dfrac{d}{dx}\left(x^2y\right)=x^2\dfrac{dy}{dx}+2xy=e^x$, so integrating both sides of this equation yields $x^2y=e^x+c$, and using the fact that $y(1)=e$, $1^2\cdot e=e^1+c$ $\Rightarrow c=0$. Therefore, $x^2y=e^x \Rightarrow y=\dfrac{e^x}{x^2}$.

20. $\dfrac{1}{3-1}\displaystyle\int_1^3\left(6x^2-4x+3\right)dx=\dfrac{1}{2}\left(2x^3-2x^2+3x\right)\Big|_1^3=\dfrac{1}{2}(54-18+9-2+2-3)=21$

21. Since $y(1)=1$, using a step size of $h=0.2$, $y(1.2)\approx 1+0.2(2\cdot 1+3\cdot 1)=2$, and $y(1.4)\approx 2+0.2(2\cdot 1.2+3\cdot 2)=3.68$.

22. The Maclaurin series for e^x, which converges to e^x for all real values of *x*, is $\displaystyle\sum_{n=0}^{\infty}\frac{x^n}{n!}$. Since the given series is equivalent to this when $x=\dfrac{1}{2}$, the sum of this series is $e^{\frac{1}{2}}=\sqrt{e}$.

23. Since $\dfrac{d}{dx}(\arctan x)=\dfrac{1}{1+x^2}$ resembles the sum of an infinite geometric series with common ratio $-x^2$, this series would converge provided $-1<-x^2<1 \Rightarrow -1<x<1$. Integrating afterward may or may not include 1 or -1 (it includes both), but that will not make the interval any longer. Since the interval of convergence has length 2, the radius of convergence is 1.

24. $f'(x) = \frac{-1}{1-x} = -1 - x - x^2 - \ldots$ for appropriate values of *x*, so $f(x)$ has power series representation $C - x - \frac{1}{2}x^2 - \frac{1}{3}x^3 - \ldots$. Plugging in $x = 0$ into both the function and its power series representation, which is centered at 0, yields $C = 0$, so the third non-zero term is $-\frac{1}{3}x^3$.

25. Since *f* is twice differentiable on all real numbers, the function and both of its first two derivatives are also continuous, meaning that the values of any of those functions and their limits are interchangeable. The limit we are looking to evaluate is in an indeterminate form of type $\frac{0}{0}$, so $\lim_{x\to 0}\frac{f(x)-f'(x)}{f'(x)-1} = \lim_{x\to 0}\frac{f'(x)-f''(x)}{f''(x)} = \frac{1-2}{2} = -\frac{1}{2}$.