Mu Alpha Theta 2019 National Convention Hustle Statistics Solutions

1. **71**. The sum of the ten numbers is 710. When you divide by the number of numbers, you get the solution.

2. **.88** or $\mathbf{\frac{22}{25}}$. When two events are independent, $P(A \cap B) = P(A)P(B)$. Plugging the values in produces $P(A \cap B) = (.7)(.8) = .56$. Therefore, $P(A \cup B) = P(A) + P(B) - P(A \cap B) = .7 + .8 - .56 = .94$. Since $P(A \cup B) = .94,\ P(A' \cap B') = .06$. Plugging into the final expression produces (.94 - .06) = .88.

3. **y = .375x + 61.25** or $\mathbf{y = \frac{3}{8}x + \frac{245}{4}}$. The formula for the equation of the line of best fit is $y - \overline{y} = r\left(\frac{s_y}{s_x}\right)(x - \overline{x})$. Plugging in produces

$$y - 80 = \left(\frac{3}{4}\right)\left(\frac{7}{14}\right)(x - 50) \rightarrow y - 80 = \frac{3}{8}(x - 50) \rightarrow y - 80 = \frac{3}{8}x - \frac{75}{4} \rightarrow y = \frac{3}{8}x + \frac{245}{4}.$$

4. **18.35** or $\mathbf{\frac{367}{20}}$. To find the mean of a discrete distribution, you find the sum of the products (X)(P(X)). Plugging the values in produces 13(.1) + 14(.2) + 16(.15) + 18(.2) + 20(.1) + 25(.25) = 1.3 + 2.8 + 2.4 + 3.6 + 2 + 6.25 = 18.35.

5. **12**. This is a two sample t test. Plugging the values into the formula assuming a positive difference produces $t = \frac{58 - 48}{\sqrt{\frac{6^2}{144} + \frac{8^2}{144}}} = \frac{10}{\sqrt{\frac{36 + 64}{144}}} = \frac{10}{\sqrt{\frac{100}{144}}} = \frac{10}{\frac{10}{12}} = 12.$

6. **5.4** or $\mathbf{\frac{27}{5}}$. The formula for a z score = $\frac{raw - mean}{sd}$. Plugging the numbers in produces $\frac{63 - 76}{5} = \frac{-13}{5} = -2.6$ for Brian and $\frac{90 - 76}{5} = \frac{14}{5} = 2.8$ for Julie. Finding the difference between the results (2.8) – (-2.6) = 5.4.

7. **.4375** or $\mathbf{\frac{7}{16}}$. Since 60% of the students are female, 40% are male. 70% of males and 60% of females do not participate in extracurricular activities. The probability that a student does not participate in extracurricular activities is (.4)(.7) + (.6)(.6) = .64. The probability of a male not participating in extracurricular activities is (.4)(.7) = .28. The overall solution is $\frac{.28}{.64} = \frac{7}{16}$.

8. **.488** or $\mathbf{\frac{61}{125}}$. This is a geometric situation. The probability that Sam is successful by the third attempt is $.2+.8(.2)+.8^2(.2)=.2+.16+.128=.488$.

9. **11.5** or $\mathbf{\frac{23}{2}}$. To find an expected value for a χ^2 test of independence, the formula is $\frac{(row\ total)(column\ total)}{(total\ of\ entire\ chart)}$. Plugging the values in for French Chemists produces $\frac{(30)(46)}{120}=\frac{23}{2}$.

10. $\mathbf{24\sqrt{2}}$. The standard deviation of 3A is 3(8) = 24. The standard deviation of 2B is 2(12) = 24. The standard deviation of (3A – 2B) = $\sqrt{24^2+24^2}=\sqrt{2(24^2)}=24\sqrt{2}$.

11. $\mathbf{\frac{15}{64}}$. The probability of rolling a prime number with a fair die is $\frac{1}{2}$ (2, 3, 5). This is a binomial situation. Therefore, the probability is ${}_6C_4\left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right)^2=15\left(\frac{1}{2}\right)^6=\frac{15}{64}$.

12. $\mathbf{\frac{2\sqrt{10}}{5}}$. The mean of the distribution is 1(.2) + 2(.1) + 3(.3) + 4(.3) + 5(.1) = .2 + .2 + .9 + 1.2 + .5 = 3. Subtracting the mean from each value, squaring the differences and multiplying those differences by their corresponding probabilities produces 4(.2) + 1(.1) + 0(.3) + 1(.3) + 4(.1) = 1.6. This is the variance of the distribution. The standard deviation is $\sqrt{1.6}=\sqrt{\frac{8}{5}}=\frac{\sqrt{40}}{5}=\frac{2\sqrt{10}}{5}$.

13. **52**. Put the numbers in increasing order. The median of the ten numbers is the average of the fifth and sixth number. The fifth number is 51 and the sixth number is 53. The average of those two numbers is the solution.

14. $\mathbf{\sqrt{14}}$. The mean of the data is 5. When you subtract the mean from each value, square the differences and add them up, you get a total of 70. You divide that total by (n-1), or 5 in this case, to get a variance of 14. The standard deviation is the square root of the variance.

15. **-55**. The formula for a residual = observed – predicted. When you plug 56 into the equation, you get a predicted final exam score of 143. Plugging the numbers in produces residual = 88 – 143 = -55.

16. $\mathbf{\frac{\sqrt{3}}{25}}$. The formula for the standard deviation of a two proportion test is $\sqrt{p(1-p)\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}$, where $p=\frac{x_1+x_2}{n_1+n_2}$. Plugging the values in produces $p=\frac{65+55}{100+100}=\frac{120}{200}=\frac{3}{5}$. Plugging p into the standard deviation produces $\sqrt{\left(\frac{3}{5}\right)\left(\frac{2}{5}\right)\left(\frac{1}{100}+\frac{1}{100}\right)}=\sqrt{\left(\frac{6}{25}\right)\left(\frac{2}{100}\right)}=\sqrt{\frac{3}{625}}=\frac{\sqrt{3}}{25}$.

17. $\mathbf{\frac{9\sqrt{91}}{20}}$. The standard deviation for a binomial distribution is $\sqrt{np(1-p)}$. Plugging the values in produces $\sqrt{81\left(\frac{65}{100}\right)\left(\frac{35}{100}\right)}=\sqrt{81\left(\frac{13}{20}\right)\left(\frac{7}{20}\right)}=\frac{9\sqrt{91}}{20}$.

18. $\mathbf{\frac{241}{3}}$. In order to go from a standard deviation of 12 to 8, you must multiply by $\frac{2}{3}$. When you multiply the original mean by $\frac{2}{3}$, you get $(62)\frac{2}{3}=\frac{124}{3}$. In order to get a new mean of 75, you must add $\frac{101}{3}$ to the result. So the transformation equation is $y=\frac{2}{3}x+\frac{101}{3}$. When you plug Matt's score in, you get $y=\frac{2}{3}(70)+\frac{101}{3}=\frac{140+101}{3}=\frac{241}{3}$.

19. **.29** or $\mathbf{\frac{29}{100}}$. Since $P(A\,|\,B')=.42=\frac{P(B'\cap A)}{P(B')}=\frac{P(B'\cap A)}{.5}\rightarrow P(B'\cap A)=.21$. Since $P(B'\cap A)=.21,\ P(A\cap B)=.17\ \textit{and}\ P(A'\cap B)=.33$. Therefore $P(A\cup B)=.21+.17+.33=.71$. Finally, $P(A'\cap B')=1-P(A\cup B)=1-.71=.29$.

20. **97**. When you put the original numbers into a Venn diagram, the totals don't match up with each class. You must subtract 17 from each of the two class numbers and redo the Venn diagram. As a result, here are the breakdowns: 11 English only, 13 English and Math, 11 English and Science, 17 all 3 classes, 18 Math only, 7 Math and Science, 20 Science only. When you add all of these results, it adds to the solution.

21. **.815**. The high score of 87 is two standard deviations above the mean. Using the 68-95-99.7 approach, this score is at the .975 percentile. The low score of 63 is one standard deviation below the mean. Using the same approach, this score is at the .16 percentile. .975 - .16 = .815.

22. $\mathbf{\frac{16}{49}}$. $m = r\left(\frac{s_y}{s_x}\right) \rightarrow \frac{20}{21} = r\left(\frac{10}{6}\right) \rightarrow r = \frac{4}{7} \rightarrow r^2 = \frac{16}{49}$.

23. **3.8** or $\mathbf{\frac{19}{5}}$. The formula for $\chi^2 = \sum \frac{(obs - \exp)^2}{\exp}$. The expected frequency for each value on the die is 10 in this case. Plugging the values in produces

$$\frac{(6-10)^2 + (11-10)^2 + (8-10)^2 + (10-10)^2 + (14-10)^2 + (11-10)^2}{10} = \frac{16+1+4+0+16+1}{10} = \frac{38}{10} = \frac{19}{5}$$

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24. **14**. In order for the game to be fair, the probability of success times the amount won must be equal for both players. The probability that Bill will win the game is $\frac{28}{52}$. Therefore, $\left(\frac{28}{52}\right)(12) = \left(\frac{24}{52}\right)X \rightarrow X = 14$.

25. **79**. Put the numbers in increasing order. The median of the data is the average of the fifth and sixth number. The fifth number is 43 and the sixth number is 63. The average between them is 53. The first quartile is the median from the minimum to the median. This value is 10. The third quartile is the median from the median to the maximum. This value is 89. The interquartile range is (third quartile – first quartile) = 89 – 10 = 79.