For all questions, answer choice "E. NOTA" means none of the above answers is correct.

(1) A particle is moving along the x-axis with position at time t given by

$$x(t) = t^3 - 9t^2 + 27t + 31.$$

At how many different times is the speed of the particle equal to 0?

A. 0 B. 1 C. 2 D. 3 E. NOTA

(2) A given bacteria culture doubles every 10 minutes, and has size 100 at time $t = 0$. How many minutes after time $t = 0$ does the bacteria culture have size 300?

A. $\frac{10\ln 2}{\ln 3}$ B. $\frac{\ln 2}{10\ln 3}$ C. $\frac{10\ln 3}{\ln 2}$ D. $\frac{\ln 3}{10\ln 2}$ E. NOTA

(3) It costs Danny a dollar to make a widget. If he sells widgets at $\$x$ each, he knows that he will be able to sell $(300 - 20x)$ of them. He also knows that he will be unable to sell any at \$15 or higher. What price $\$x$ should he set in order to maximize his profit?

A. \$7.00 B. \$7.50 C. \$8.00 D. \$8.50 E. NOTA

(4) A plane flies parallel to the ground on a trajectory toward an observer (technically, a point directly above the observer). The plane flies at a constant height of 4 miles above the ground with constant velocity of 8 miles per minute, and at a particular instant, the plane is 5 miles away from the observer. At that instant, at what rate does the angle at which the observer has to look up to see the plane increase (in radians per minute)?

A. $4/25$ B. $8/25$ C. $16/25$ D. $32/25$ E. NOTA

(5) A parabola has roots $x = 2/3, 17/3$, and goes through the point $(19/6, 3)$. What is the area bounded by this parabola and the x-axis?

A. 10 B. 11 C. 12 D. 13 E. NOTA

(6) Amy and Bob are sharing a pizza. First, Amy takes half the pizza and eats it. Then Bob takes half of what's left and eats it. Then Amy takes half of what remains and eats it. They continue in this fashion until (after an infinite amount of time) there is no pizza left. What fraction of the pizza did Bob eat?

A. $1/5$ B. $1/4$ C. $1/3$ D. $1/2$ E. NOTA

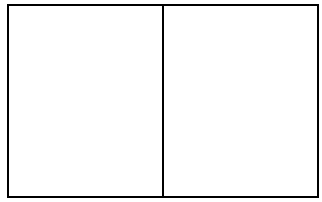
(7) A car travels along a highway for 7 hours from time $t = 0$ to $t = 7$. We observe the instantaneous velocities of the car at some points:

t (hrs)	0	1	2	3	4	5	6
v (mph)	40	60	50	40	50	50	20

Use a left-handed Riemann sum with 7 equal sub-intervals to estimate the distance the car traveled in this 7-hour period.

A. 310 B. 320 C. 620 D. 640 E. NOTA

(8) A farmer wishes to make a rectangular pen with two sub-divisions as depicted in the diagram below:



If the farmer only has 120 feet of fencing to make this pen, what is the maximum possible area (in square feet) that could be enclosed by the pen?

A. 450 B. 500 C. 600 D. 900 E. NOTA

(9) The region bounded by $y = \sin x$, the x-axis, and the lines $x = 0$, $x = \pi$ is rotated about the x-axis. What is the volume of the resulting solid?

A. $\frac{\pi}{4}$ B. $\frac{\pi}{2}$ C. $\frac{\pi^2}{4}$ D. $\frac{\pi^2}{2}$ E. NOTA

(10) A rope with mass density of 2 lbs/ft has length 10 feet. It is currently coiled up on the ground. A crane grabs one end of the rope and lifts it up 20 feet. How much work (in ft-lbs) did the crane do on the rope?

A. 150 B. 200 C. 300 D. 400 E. NOTA

(11) Water is poured into an upright cone at a rate of 3π ml/sec. The cone has base radius 4 cm and height 10 cm. When the height of the water in the cone is 5 cm, how quickly (in cm/sec) is the water level increasing?

A. $\frac{3}{16}$ B. $\frac{3}{8}$ C. $\frac{3}{4}$ D. $\frac{3}{2}$ E. NOTA

(12) Kwesi travels along the x-axis. He leaves the origin at time $t = 0$ and has velocity $2t + 2$ at time t. Philip leaves the origin at time $t = 2$ and will maintain a constant velocity v. What is the least v so that Philip will eventually catch up with Kwesi?

A. $6 + 4\sqrt{2}$ B. $4 + 6\sqrt{2}$ C. $7 + 5\sqrt{2}$ D. $5 + 7\sqrt{2}$ E. NOTA

(13) Define how far away a point is from a curve to be the shortest distance between that point and a point on the curve. Consider the point on $y = \sqrt[3]{x^3 + 1}$ that is farthest away from the line $y = x$; how far is it?

A. $\sqrt[3]{2}$ B. $\sqrt[3]{4}$ C. $\sqrt[6]{16}$ D. $\sqrt[6]{32}$ E. NOTA

(14) The parabola $f(x) = x^2 + bx + c$ is tangent to the lines $y = x$ and $y = -2x$. Compute $b + c$.

A. $\frac{1}{16}$ B. $\frac{1}{8}$ C. $\frac{1}{4}$ D. $\frac{1}{2}$ E. NOTA

(15) Adam is traveling along a line. He starts from rest at the origin at time $t = 0$, and for all times $0 < t < T$, his velocity is $v(t) = t^2$. However, at time T, he gets tired and stops moving. Time T is selected uniformly at random from the real numbers between 0 and 5. What is the expected value of the distance Adam travels?

A. $\frac{125}{48}$ B. $\frac{125}{24}$ C. $\frac{125}{12}$ D. $\frac{125}{6}$ E. NOTA

(16) Consider the region in the xy-plane characterized by $x^2 + y^2 < 1$ and $y > 0$. What is the x-coordinate of the centroid of this region?

A. 0 B. $\frac{2}{3}$ C. $\frac{3}{4}$ D. $\frac{4}{5}$ E. NOTA

(17) A particle moves along the x-axis with velocity $v(t) = x$ and starts at position $x(0) = 1$. At what x position will the particle be at time $t = 1$?

A. e B. 1 C. $1/2$ D. $3/2$ E. NOTA

(18) The curves $f(x) = x^3 + 5x$ and $g(x) = 4x^2 + 2$ intersect only at $x = 1$ and $x = 2$. Consider an estimate of the area bounded by these two curves given by Simpson's rule with 4 equal sub-intervals. By how much does it overestimate the true area bounded by these two curves?

A. 0 B. $\frac{1}{3}$ C. $\frac{1}{4}$ D. $\frac{5}{12}$ E. NOTA

(19) A cylinder is inscribed inside a cone with volume V. What is the maximum possible volume of the cylinder?

A. $\frac{V}{3}$ B. $\frac{2V}{3}$ C. $\frac{4V}{9}$ D. $\frac{8V}{27}$ E. NOTA

(20) An ellipse has major axis of length 6 and minor axis of length 4. What is the maximum possible area of a hexagon inscribed in this ellipse?

A. $6\sqrt{3}$ B. $9\sqrt{3}$ C. $24\sqrt{3}$ D. $36\sqrt{3}$ E. NOTA

(21) What is the average distance from a point in a sphere of radius R to the center of the sphere?

A. $\frac{R}{2}$ B. $\frac{2R}{3}$ C. $\frac{3R}{4}$ D. $\frac{4R}{5}$ E. NOTA

(22) What is the slope of the tangent line to $y = x^2\sin(x)$ at $x = \pi$?

A. π B. $-\pi$ C. π^2 D. $-\pi^2$ E. NOTA

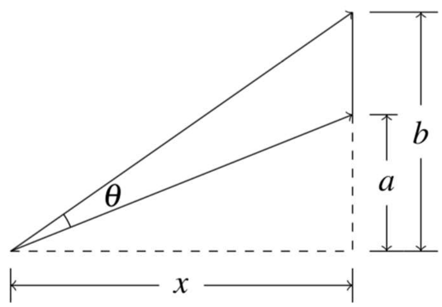
(23) Consider a negligibly thin, inelastic wire with length L and constant linear density stretched out along the parabola $y = x^2$ from the point $(0,0)$ to the point $(3,9)$. What is the x-coordinate of the center of mass of this wire?

A. $\frac{3}{2}$ B. $\frac{37\sqrt{37}-1}{12L}$ C. $\frac{9}{4}$ D. $\frac{41\sqrt{41}-1}{12L}$ E. NOTA

(24) What is the volume of a solid with a circular base of radius R and equilateral triangle cross sections perpendicular to the base?

A. $\frac{4\sqrt{3}R^3}{3}$ B. $\frac{\sqrt{3}R^3}{3}$ C. $\frac{\sqrt{3}R^3}{4}$ D. $4\sqrt{3}R^3$ E. NOTA

(25) You are standing a horizontal distance x away from a picture that is a units above your eye level and $b - a$ units tall. If you want to maximize the viewing angle θ, at what distance x should you stand?



A. $\frac{a+b}{2}$ B. $\sqrt{ab}$ C. $\frac{1}{\frac{1}{a}+\frac{1}{b}}$ D. $\frac{2}{\frac{1}{a}+\frac{1}{b}}$ E. NOTA

(26) Kim and Ellen start at the same point. They then simultaneously start walking away from this point. Each is on a linear path, and their paths form a $60°$ angle at the origin. Kim walks at 5 mph and Ellen walks at 4 mph. Exactly one hour after they start walking, at what speed is the linear distance between them increasing (in mph)?

A. $\frac{29}{3}$ B. $\frac{29\sqrt{3}}{2}$ C. $\frac{31}{3}$ D. $\frac{31\sqrt{3}}{2}$ E. NOTA

(27) Consider the curves $f(x) = x^n$, $g(x) = x^{n+1}$, and $h(x) = x^{n+2}$, where n is a positive integer. Let A be the area bounded by f and g, and let B be the area bounded by g and h. What is

$$\lim_{n\to\infty} \frac{A}{B}?$$

A. 1 B. 2 C. 3 D. ∞ E. NOTA

(28) Evaluate:

$$\lim_{x\to 0} \frac{\sin(2x)}{3x}$$

A. 0 B. $\frac{1}{3}$ C. $\frac{2}{3}$ D. DNE E. NOTA

(29) Suppose that at time t a particle is at position $x(t) = -t^3 + 6t^2 - 11t$. How many times does this particle change direction?

A. 0 B. 1 C. 2 D. 3 E. NOTA

(30) What is the maximum value of

$$90\sin(x) + 120\cos(x)$$

on its domain?

A. 120 B. 130 C. 140 D. 150 E. NOTA