Mu Applications Answers

1. B
2. C
3. C
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5. A
6. C
7. A
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25. B
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Mu Applications Solutions

(1) B The velocity of the particle is given by the derivative,

$$\boldsymbol{v(t) = x'(t) = 3(t-3)^2}$$

which has one zero at $\boldsymbol{t=3}$. Since speed is the absolute value of velocity, in order for the speed to be zero, the velocity must be zero. Thus, the answer is that there is only $\mathbf{1}$ such time, namely $\boldsymbol{t=3}$.

(2) C From the information, we can identify that the size of the population at time t must be given by,

$$100 \cdot 2^{t/10}$$

and we wish to solve the equation

$$100 \cdot 2^{t/10} = 300 \Longrightarrow 2^{t/10} = 3 \Longrightarrow \frac{t}{10} = \frac{\ln 3}{\ln 2} \Longrightarrow t = \boxed{\frac{10 \ln 3}{\ln 2}}.$$

(3) C Danny's profit per unit will be the amount he sells the widget at minus the cost of producing the widget, or

$$\text{unit profit} = x - 1$$

and since he sells $300 - 20x$ of them so long as $x < 15$, he makes a total profit of,

$$\text{profit} = (x-1)(300-20x)$$

which is a parabola that opens downward with vertex at $(8{,}980)$ meaning that the optimal x is $x = \$8.00$.

(4) D The angle that the plane makes at any given point in time is,

$$\theta = \tan^{-1}\left(\frac{h}{x}\right)$$

where h is the height of the plane, and x is the horizontal component of the distance from the plane to the observer. In particular, h stays constant, and x is decreasing at a rate of 8, which allows us to differentiate,

$$\frac{d\theta}{dt} = -\frac{1}{1+\left(\frac{h}{x}\right)^2} \cdot \frac{h}{x^2} \cdot \frac{dx}{dt} = -\frac{h \cdot v}{x^2 + h^2}$$

where v is the velocity of the plane. Thus, we can simplify,

$$\frac{8 \cdot 4}{16+9} = \boxed{\frac{32}{25}}$$

(5) A One way to solve this problem is to find the equation for the parabola, and then integrate from $2/3$ to $17/3$. Alternatively, we may consider the parabola

$$f(x) = 4x(1-x)$$

which bounds an area of,

$$\int_0^1 4x(1-x)\,dx = 2x^2 - \frac{4}{3}x^3\Big|_0^1 = 2 - \frac{4}{3} = \frac{2}{3}.$$

This region will be the same as a stretched version of the region we wish to find the area of. In particular, we have stretched it by a factor of,

$$\frac{17}{3}-\frac{2}{3}=5$$

in the x-direction, and by a factor of 3 in the y-direction. Thus, the area of this region will equal,

$$\frac{2}{3}\cdot 5\cdot 3=\boxed{10}.$$

(6) C The amount of the pizza that Bob eats is given by the infinite series,

$$\sum_{i=1}^{\infty}\frac{1}{4^i}$$

which we can evaluate noting that the ratio is $1/4$ and the initial term is $1/4$ as

$$\frac{a_0}{1-r}=\frac{1/4}{1-1/4}=\frac{2}{6}=\boxed{\frac{1}{3}}.$$

(7) A The sub-interval width is 1 hour, which means that we can simply add up the observations to estimate the distance travelled as,

$$40\ +\ 60\ +\ 50\ +\ 40\ +\ 50\ +\ 50\ +\ 20\ =\ 310 \text{ miles.}$$

(8) C Suppose that the width of the pen is x and the height is y. This requires

$$2x+3y$$

feet of fencing, and we therefore have the constraint that

$$2x+3y=120.$$

We wish to maximize the total area inside the pen, or the product xy. We can then solve for x

$$2x+3y=120\Longrightarrow x=60-\frac{3}{2}y$$

and maximize the univariate function

$$y\left(60-\frac{3}{2}y\right)$$

which gives us that $y=20\Longrightarrow x=30$ and the maximal possible area is $xy=\boxed{600}$.

(9) D We can find the volume of the region by integrating,

$$\int_0^{\pi}\pi\sin^2 x\ dx=\int_0^{\pi}\pi\frac{1-\cos(2x)}{2}dx=\frac{\pi^2}{2}.$$

(10) C The work that the crane does on the rope is equal to,

$$\int_{10}^{20}2x\,dx=\ \ x^2\Big|_{10}^{20}=400-100=\boxed{300}.$$

(11) C The water level will increase by a rate equal to the rate of change of the volume of the water divided by the surface area of the water currently in the cone. Because the cone is filled to a height of 5 cm, this means that the current surface area of the water is a circle with radius 2 cm, and thus area 4π cm^2. This makes the rate of change of the water level equal to,

$$\frac{3\pi}{4\pi} = \boxed{\frac{3}{4}}.$$

(12) A In order to find the least possible v, we must have that Philip just barely catches up with Kwesi, which means that their paths can only intersect at one point. The path that Kwesi takes is given by,

$$x_{\text{Kwesi}}(t) = t^2 + 2t$$

and the path that Philip takes is given by,

$$x_{\text{Philip}}(t) = v(t-2).$$

Thus, in order for these to intersect at only one point, we must have that,

$$t^2 + 2t - v(t-2) = t^2 + (2-v)t + 2v$$

is a square, which requires the discriminant to equal 0,

$$b^2 - 4ac = 0 \implies (2-v)^2 - 8v = 0 \implies v = 6 \pm 4\sqrt{2}$$

and in order for the intersection to happen for positive time, we must have that,

$$v = \boxed{6 + 4\sqrt{2}}.$$

(13) E We can find the point on the curve that is furthest away from the line. At this point, the slope of the curve must be 1, and so we can solve,

$$\frac{d}{dx}\sqrt[3]{x^3+1} = 1 \implies x = -\frac{1}{\sqrt[3]{2}}.$$

This corresponds to the point $\left(\frac{1}{\sqrt[3]{2}}, \sqrt[3]{\frac{3}{2}}\right)$. The distance from this point to $y = x$ is $\frac{\left|\frac{1}{\sqrt[3]{2}} - \sqrt[3]{\frac{3}{2}}\right|}{\sqrt{2}} = \frac{\sqrt[3]{3}-1}{\sqrt[3]{2}\sqrt{2}}$.

(14) A The function $y = f(x) - x$ must be tangent to $y = 0$ and $y = -3x$. Since the parabola opens upward and has leading coefficient 1, this means that,

$$f(x) - x = (x-a)^2.$$

In order for this to be tangent to $y = -3x$, there must exist some x such that

$$(x-a)^2 = -3x \text{ and } 2(x-a) = -3$$

which solves to give us that $a = 3/4$, such that

$$f(x) - x = \left(x - \frac{3}{4}\right)^2 \implies f(x) = x^2 - \frac{1}{2}x + \frac{9}{16}.$$

(15) C The expression for the distance that Adam travels can be simplified to

$$\int_0^T t^2\,dt = \frac{T^3}{3},$$

which we want to average when T goes from 0 to 5. We can do this through an integral, to get

$$\frac{1}{5}\int_0^5 \frac{x^3}{3}\,dx = \boxed{\frac{125}{12}}.$$

(16) A This region is symmetric, which means that the centroid will be on the axis of symmetry, and therefore the x value will be 0.

(17) A This is a differential equation $x' = x$ which is solved by $x(t) = e^t$ given the initial condition. Evaluating at $t = 1$ gives us our desired answer of e.

(18) A Simpson's rule perfectly approximates the integration of cubics, and thus its error is $\boxed{0}$.

(19) C We can without loss of generality assume that the cone has radius 1 and height 1 because any linear transformation of these values will affect the area of any cylinder by the same factor. If we say that the cylinder has radius r, it must thus have height $(1 - r)$, and the area of the cylinder will be given by,

$$\pi r^2 h = \pi r^2(1 - r)$$

which is maximized when

$$r = \frac{2}{3}.$$

This gives us that the volume of the cylinder divided by the volume of the cone is,

$$\frac{(2/3)^2(1/3)}{1/3} = \boxed{\frac{4}{9}}.$$

(20) B We can note that the ellipse is simply a linear transformation of a unit circle, and thus the maximization problem is simply a scaled version of the same problem for the unit circle. By symmetry, the hexagon of maximal area inscribed in the unit circle is the normal hexagon with area,

$$\frac{\sqrt{3}}{4}\cdot 6 = \frac{3\sqrt{3}}{4}.$$

Since we have scaled the problem down by a factor of $3 \cdot 2 = 6$, our answer is,

$$\frac{3\sqrt{3}}{4}\cdot 6 = \boxed{9\sqrt{3}}.$$

(21) C We can integrate the density of the sphere (which is proportional to the square of the radius) times the radius in order to get the (weighted) average distance from a point in a sphere to the center as,

$$\bar{d} = \frac{\int_0^R r^3\,dr}{\int_0^R r^2\,dr} = \boxed{\frac{3R}{4}}.$$

(22) D We can differentiate to get

$$y' = x^2\cos(x) + 2x\sin(x)$$

and evaluating at $x = \pi$ gives us,

$$= \boxed{-\pi^2}.$$

(23) B The x-coordinate of the center of mass of this wire will be the average x-coordinate of all points on the wire. This means that we can integrate,

$$\bar{x} = \frac{1}{L}\int_0^3 x\,dl$$

and we can rewrite

$$dl = \sqrt{1 + f'(x)^2}\,dx = \sqrt{1 + 4x^2}\,dx$$

which makes the integral

$$\frac{1}{L}\int_0^3 x\sqrt{1 + 4x^2}\,dx = \frac{37\sqrt{37} - 1}{12L}.$$

(24) A We recall that the area of an equilateral triangle is given by,

$$\frac{\sqrt{3}s^2}{4}$$

and then we can integrate

$$\int_{-R}^{R} \sqrt{3}(R^2 - r^2)\,dr = \boxed{\frac{4\sqrt{3}R^3}{3}}.$$

(25) B The viewing angle is given by,

$$\theta = \tan^{-1}(b/x) - \tan^{-1}(a/x).$$

We can maximize this by differentiating with respect to x and setting it equal to 0, which gives us,

$$\frac{a}{a^2 + x^2} - \frac{b}{b^2 - x^2} = 0 \Longrightarrow a(b^2 + x^2) = b(a^2 + x^2) = x = \sqrt{ab}$$

which we confirm is indeed a maximum by checking concavity.

(26) E We can label $x(t)$ and $y(t)$ as the distances from the origin for Kim and Ellen at time t respectively. $z(t)$ will refer to the distance between them at time t. The law of cosines tells us that,

$$z^2 = x^2 + y^2 - 2xy\cos 60°.$$

Thus, we can now implicitly differentiate to get that,

$$2zz' = 2xx' + 2yy' - (xy' + x'y)$$

where we can plug in the values at this particular instance,

$$2\sqrt{21}z' = 25 + 16 - 12 \Longrightarrow z' = \frac{29\sqrt{21}}{42}.$$

(27) A First, we can note that all of these curves will intersect at $(0,0)$ and $(1,1)$. Thus, we can find expressions for A and B by integrating on this range which gives us that,

$$A = \frac{1}{n+1} - \frac{1}{n+2}$$

and

$$B = \frac{1}{n+2} - \frac{1}{n+3}.$$

This tells us that,

$$\frac{A}{B} = \frac{n+3}{n+1}$$

and since we are taking the limit as n goes to 0, we need only look at the coefficients of the highest powers of n to conclude that the limit is 1.

(28) C Recall that the limit as x goes to 0 of $\sin(x)/x$ is 1 which we can see by L'Hospital. Using L'Hospital here tells us that this limit is $2/3$.

(29) C We can differentiate the position function and find that its discriminant is positive such that it has two zeros. Thus, the particle changes direction twice.

(30) D We can first of all identify this as

$$30(3\sin x + 4\cos x)$$

which further simplifies to

$$150\sin(x + \tan^{-1}(4/3)).$$

At this point, it becomes evident that the maximum value this can take is 150 when the term inside of the sin is equal to $\pi/2$ for example.