For all questions below, the answer E. NOTA means "None of these answers".

1. Congruent squares are cut out of each corner of a sheet of paper that is 10 inches wide and 16 inches long. The flaps are then folded up to form a box without a top. In cubic inches, what is the maximum volume that this box can hold?

A. 72 B. 144 C. 224 D. 288 E. NOTA

2. Consider the unbounded region below $y = e^{2x}$ and $y = e^{-2x}$, and above $y = 0$ over the interval $(-\infty, \infty)$. What is the area of this region?

A. $\frac{1}{4}$ B. $\frac{1}{2}$ C. 1 D. $2e$ E. NOTA

3. A small sphere is inscribed in a cube, which is inscribed in a larger sphere. What is the ratio of the surface area of the larger sphere to the surface area of the smaller sphere?

A. 2:1 B. 3:1 C. 4:1 D. 12:1 E. NOTA

4. A rectangle is inscribed in the semicircle bound between $y = \sqrt{32 - x^2}$ and $y = 0$ so that two of its sides are parallel to the y-axis. What is the largest possible area of this rectangle?

A. 24 B. 30 C. 32 D. 48 E. NOTA

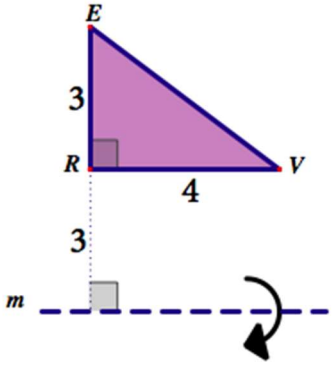
5. Find the area that lies outside the curve $r = 3$ but inside the curve $r = 2 + 2\sin(\theta)$.

A. $3\sqrt{3} - \pi$ B. $\frac{9\sqrt{3}}{2} + \frac{3\pi}{2}$ C. $9\sqrt{3} - 2\pi$ D. $\frac{9\sqrt{3}}{2} - \pi$ E. NOTA

6. The tangent line to a point on the curve $y = 3 - x^2$ in the first quadrant intersects the axes at two points. What is minimum area enclosed by the triangle whose vertices are these two points and the origin?

A. 3 B. 4 C. 6 D. 12 E. NOTA

7. Base $\overline{RV}$ is 3 units away from line m and is parallel to m. In cubic units, what is the volume of the solid formed when ΔREV and its interior are revolved around axis m?



A. 36π B. 48π C. 64π D. 84π E. NOTA

For #8 and #9 consider the region *R* bounded by $f(x) = \sin(x)$, $g(x) = \cos(x)$, for $\frac{\pi}{4} \le x \le \frac{5\pi}{4}$.

8. Find the area of the region *R*.

A. $\sqrt{2}$ B. $2\sqrt{2}$ C. $2\sqrt{3}$ D. $4\sqrt{2}$ E. NOTA

9. Which of the following represents the volume when *R* is revolved about *y* = -1?

A. $\pi \int_{\frac{\pi}{4}}^{\frac{5\pi}{4}} ((\sin x + 1)^2 - (\cos x + 1)^2)\, dx$ B. $\pi \int_{\frac{\pi}{4}}^{\frac{5\pi}{4}} ((\sin x)^2 - (\cos x)^2 - 1) dx$

C. $\pi \int_{\frac{\pi}{4}}^{\frac{5\pi}{4}} ((\sin x - \cos x)^2 - 1)\, dx$ D. $\pi \int_{\frac{\pi}{4}}^{\frac{5\pi}{4}} ((\sin x - 1)^2 - (\cos x - 1)^2) dx$ E. NOTA

10. Find the area of the region bounded by $|x| + |y| = 6$.

A. 6 B. 9 C. 36 D. 72 E. NOTA

11. Find the volume of the solid when the region bounded by $y = e^{-x}, y = 0, x = -1, x = 0$ is rotated about the line $x = 1$.

A. $2\pi e$ B. $2\pi e^2$ C. $2\pi(e - 2)$ D. $2\pi(2e - 1)$ E. NOTA

12. Find the area bounded by $y = \frac{2}{x^2+4x+3}, y = 0, x = 0, x = 2$.

A. $\ln(\frac{9}{5})$ B. $\ln(\frac{6}{5})$ C. $\ln(\frac{25}{9})$ D. $\ln(\frac{16}{9})$ E. NOTA

13. A wire of length *L* is cut into two lengths, and these two lengths are bent to form two separate squares. In terms of *L,* what is the minimum possible sum of the squares' areas?

A. $\frac{L^2}{64}$ B. $\frac{L^2}{32}$ C. $\frac{L^2}{16}$ D. $\frac{L^2}{4}$ E. NOTA

14. Using Trapezoidal rule with 4 equal subintervals, approximate the area bounded between $y = \sin^2 x$ and $y = 0$ for $0 \le x \le \pi$.

A. $\frac{\pi}{4}$ B. $\frac{\pi}{2}$ C. $\frac{3\pi}{4}$ D. π E. NOTA

15. Find the *volume* of a regular octahedron whose edges are of length 6.

A. 72 B. $72\sqrt{2}$ C. 144 D.216 E. NOTA

16. Find the *surface area* of a regular octahedron whose edges are of length 6.

A. $36\sqrt{3}$ B. $72\sqrt{2}$ C. $72\sqrt{3}$ D. $144\sqrt{3}$ E. NOTA

17. Consider the region *R* bounded by the *x*-axis, $x = 0$, $x = 2$ and $y = x^3$. The line $x = a$ divides *R* into two regions of equal area. Find the value of *a*.

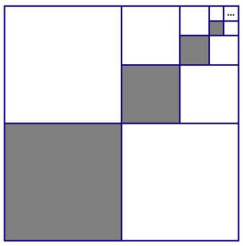
A. $a = 1$ B. $a = 2^{\frac{1}{4}}$ C. $a = 2^{\frac{1}{2}}$ D. $a = 2^{\frac{3}{4}}$ E. NOTA

18. Which integral represents the total area enclosed by the polar graph $r^2 = 9\cos(2\theta)$?

A. $9\int_0^{\frac{\pi}{4}} \cos(2\theta)\, d\theta$ B. $18\int_0^{\frac{\pi}{4}} \cos(2\theta)\, d\theta$ C. $36\int_0^{\frac{\pi}{2}} \cos(2\theta)\, d\theta$

D. $162\int_0^{\frac{\pi}{4}} \cos^2(2\theta)\, d\theta$ E.NOTA

19. The largest square *S* is divided into four squares of equal area, and the bottom left square is shaded. The congruent square diagonally opposite this shaded square is divided into four squares of equal area, and the bottom left square is shaded. This process is repeated infinitely. What fraction of the area of *S* is shaded?



A. $\frac{1}{4}$ B. $\frac{1}{3}$ C. $\frac{2}{5}$ D. $\frac{1}{2}$ E. NOTA

20. Find the volume of a solid whose base is bounded by the circle $x^2 + y^2 = r^2$ and whose cross-sections perpendicular to the *x*-axis are squares.

A. $\frac{4r^3}{3}$ B. $\frac{8r^3}{3}$ C. $4r^3$ D. $\frac{16^{\ 3}}{3}$ E. NOTA

21. Let *S* be a square where each side has length 3 units. Let *T* be the set of points in the plane exterior to the square exactly 1 unit from the nearest point on *S*. What is the total area enclosed by *T*?

A. $9 + \pi$ B. $12 + \frac{\pi}{4}$ C. $12 + \pi$ D. $21 + \pi$ E. NOTA

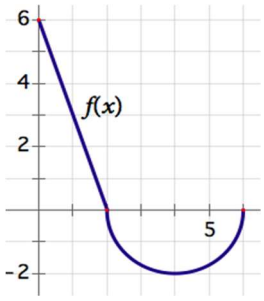
22. A $3 \times 3 \times 3$ cube consists of 27 unit cubes. An additional unit cube is placed on top of the center cube on the top face. In square units, what is the surface area of this new figure?

A. 56 B. 57 C. 58 D. 60 E. NOTA

23. For some positive value *k,* the area of the region(s) bounded between $y = |\cos x|$ and the *x*-axis for $-k \le x \le k$ is 6. Find the value of *k*.

A. $\frac{\pi}{2}$ B. $\frac{2\pi}{3}$ C. $\frac{4\pi}{3}$ D. 2π E. NOTA

24. The graph shows function $f(x)$ on the interval $[0, 6]$, where the scale is 1 unit. The graph consists of a segment connecting $(0, 6)$ and $(2, 0)$ and a semicircle of radius 2. Determine $2\int_0^6 (f(x) + 2)dx$.



A. $18 - 2\pi$ B. $36 - 2\pi$ C. $36 - 4\pi$ D. $16 + 4\pi$ E. NOTA

25. What is the maximum area of an isosceles triangle with legs of length 8?

A. 16 B. 32 C. 48 D. 64 E. NOTA

26. Consider a rectangle where the length is decreasing at a rate of 4 cm/sec while the width is increasing at a rate of 2 cm/sec. What is true about the area at the instant that the length is 8 cm and the width is 5 cm?

A. The area is increasing at 4 cm^2/sec B. The area is decreasing at 4 cm^2/sec
C. The area is increasing at 22 cm^2/sec D. The area is decreasing at 22 cm^2/sec
E. NOTA

27. A closed rectangular prism with a square base has a volume of V cubic units. What is the minimum surface area of the prism in terms of V?

A. $6V^{\frac{2}{3}}$ B. $6V^{\frac{1}{3}}$ C. $12V^{\frac{2}{3}}$ D. $12V^{\frac{4}{3}}$ E. NOTA

28. A spherical balloon is being deflated at a rate of $\pi \frac{ft^3}{min}$. Find the rate of change of the radius at the instant that the radius is $1\ ft$.

A. $\frac{1}{2}\frac{ft}{min}$ B. $-1\frac{ft}{min}$ C. $-\frac{1}{2}\frac{ft}{min}$ D. $-\frac{1}{4}\frac{ft}{min}$ E. NOTA

29. Determine the surface area when the curve $f(x) = \sqrt{16 - x^2}$ on $-4 \le x \le 4$ is revolved about the x-axis.

A. 32π B. 48π C. 64π D. 80π E. NOTA

30. From the origin, the vectors $\vec{u} =< 3,2,1 >$, $\vec{v} =< -1,3,0 >$, and $\vec{w} =< 2,2,5 >$ form a parallelepiped. What is its volume?

A. 28 B. 43 C. 47 D. 54 E. NOTA