The acronym "NOTA" stands for "None Of The Above".

1. If p, q, and r are non-zero differentiable functions, then the derivative of $\frac{p}{qr}$ is

A. $\frac{p'qr-pq'r-pqr\prime}{qr}$ B. $\frac{p'qr-pq'r-pqr\prime}{q^2r^2}$ C. $\frac{p'qr+pq'r+pqr\prime}{q^2r^2}$ D. $\frac{-p'qr+pq'r+pqr\prime}{q^2r^2}$ E. NOTA

2. Find the Maclaurin series for $f(x) = \frac{x}{4-2x}$.

A. $\sum_{n=0}^{\infty} \frac{x^n}{2^n}$ B. $\sum_{n=0}^{\infty} \frac{x^{n+1}}{2^{n+2}}$ C. $\sum_{n=0}^{\infty} \frac{x^{n+1}}{2^{n+1}}$ D. $\sum_{n=0}^{\infty} \frac{x^{n+2}}{2^{n+1}}$ E. NOTA

3. If $x + 5y = 20$ is normal to the graph of f at the point $(1,2)$, then $f'(1) =$

A. -5 B. 5 C. $-\frac{1}{5}$ D. $\frac{1}{5}$ E. NOTA

4. The area of the enclosed region bounded by the polar graph of $r = \sqrt{3 + \sin(\theta)}$ is given by the integral

A. $\int_0^{\pi} \sqrt{3 + \sin(\theta)}d\theta$

B. $\int_0^{\pi} 3 + \sin(\theta)d\theta$

C. $2\int_0^{\frac{\pi}{2}} 3 + \sin(\theta)d\theta$

D. $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 3 + \sin(\theta)d\theta$

E. NOTA

5. Let $g(x) = \frac{1}{f^{-1}(x)}$. Given the following data, determine $g'(4)$.

x	0	1	2	3	4
$f(x)$	-2	-1	2	4	6
$f'(x)$	$\frac{1}{2}$	$\frac{2}{3}$	1	$\frac{4}{3}$	$\frac{5}{3}$

A. $-\frac{1}{12}$ B. $-\frac{1}{15}$ C. $\frac{1}{12}$ D. $\frac{1}{15}$ E. NOTA

6. Evaluate $\lim_{x\to\infty} \left(\ln(2) - \ln(x) - \ln\left(tan^{-1}\left(\frac{5}{x}\right)\right)\right)$.

A. $\frac{2}{5}$ B. $\ln\left(\frac{5}{2}\right)$ C. $\ln\left(\frac{2}{5}\right)$ D. DNE E. NOTA

7. Find the equation of the line tangent to the following curve at $(0,2)$:

$$x = \ln(t), y = 1 + t^2$$

A. $y = \frac{x}{2} + 2$ B. $y = 2x - 2$ C. $y = -2x + 2$ D. $y = 2x + 2$ E. NOTA

8. The solution of the differential equation $\frac{dy}{dx} = \frac{xy\sin(x)}{y+1}$ passes through $(0, 1)$ and is a function. Solve for y.

A. $y + \ln y = -x\cos(x) + \sin(x)$.
C. $y + \ln y = -x\cos(x) + \sin(x) + 1$.
E. NOTA
B. $y + \ln y = -x\cos(x) + \sin(x) - 1$.
D. $y + \ln y = x\cos(x) - \sin(x) + 1$.

9. Compute

$$\frac{d}{dx}\left[\frac{3(x^3+1)^4\arctan(x)}{x^{\frac{1}{3}}\cdot\sqrt{x^2+1}}\right]\Bigg|_{x=1}$$

A. $24 + 62\pi$
B. $12\sqrt{2} + 37\sqrt{2}\pi$
C. $12\sqrt{2} + 35\sqrt{2}\pi$
D. $12\sqrt{2} + 31\sqrt{2}\pi$
E. NOTA

10. Which of the following functions has the Maclaurin series below?

$$\sum_{n=0}^{\infty}\frac{x^n}{n+1}$$

A. $-\frac{\ln(1-x)}{x}$
B. $\frac{\ln(1-x)}{x}$
C. $\ln(1-x)$
D. $-\ln(1-x)$
E. NOTA

11. Find the Maclaurin series for $\sin^2(x)$.

A. $\sum_{n=1}^{\infty}(-1)^n\left(\frac{2^{2n+1}}{(2n)!}\right)x^{2n}$
C. $\sum_{n=0}^{\infty}(-1)^{n+1}\left(\frac{2^{2n+1}}{(2n)!}\right)x^{2n}$
E. NOTA
B. $\sum_{n=1}^{\infty}(-1)^{n+1}\left(\frac{2^{2n}}{(2n)!}\right)x^{2n}$
D. $\sum_{n=1}^{\infty}(-1)^{n+1}\left(\frac{2^{2n+1}}{(2n)!}\right)x^{n}$

12. The doubling period of a bacterial population is 10 minutes. At time $t_1 = 45$ minutes, the bacterial population was 100. Determine the population at time $t_2 = 75$ minutes.

A. 200
B. 400
C. 600
D. 800
E. NOTA

13. Determine the arc length of the following curve: $y = \frac{1}{5}\ln(2\cos(5x))$, $\frac{\pi}{30} \le x \le \frac{\pi}{15}$

A. $\frac{1}{10}\ln\left[\frac{4}{3}+\frac{4}{\sqrt{3}}\right]$
B. $\frac{1}{10}\ln\left[\frac{7}{3}+\frac{4}{\sqrt{3}}\right]$
C. $\frac{1}{5}\ln\left[\frac{7}{3}-\frac{4}{\sqrt{3}}\right]$
D. $\frac{1}{10}\ln\left[\frac{11}{3}\right]$
E. NOTA

14. Evaluate

$$\int \frac{(\ln(\ln x))^2}{x} dx$$

A. $x(\ln x)^2 + 2x\ln x + 2x + C$

B. $x(\ln x)^2 - 2x\ln x + 2x + C$

C. $(\ln x)(\ln(ln\, x))^2 + 2(\ln x)(\ln(ln\, x)) + 2\ln x + C$

D. $(\ln x)(\ln(ln\, x))^2 - 2(\ln x)(\ln(ln\, x)) + 2\ln x + C$

E. NOTA

15. Evaluate

$$\int_1^4 e^{\sqrt{x}} dx$$

A. $2e^2$ B. $6e^2$ C. $2e^4$ D. $6e^4$ E. NOTA

16. Which integral gives the length of the curve defined by the following parametric equations?

$$x = t + \sqrt{t}, y = t - \sqrt{t}, 0 \le t \le 1$$

A. $\int_0^1 \sqrt{2 + \frac{1}{2t} - \frac{2}{\sqrt{t}}}\, dt$

C. $\int_0^1 \sqrt{1 + \frac{1}{2t}}\, dt$

E. NOTA

B. $\int_0^1 \sqrt{2 + \frac{1}{2t} + \frac{2}{\sqrt{t}}}\, dt$

D. $\int_0^1 \sqrt{2 + \frac{1}{2t}}\, dt$

17. Consider the region in the xy-plane characterized by $x^2 + y^2 < 1$ and $y > 0$. What is the y-coordinate of the centroid of this region?

A. $\frac{1}{2}$ B. $\frac{2}{3}$ C. $\frac{3}{4}$ D. $\frac{4}{5}$ E. NOTA

18. A wheel of radius R is rolling without slipping on flat ground. At time $t = 0$, a small light is placed at the bottom of the wheel, and at time $t = 2\pi$, the wheel has undergone one full rotation so that the light is once again at the bottom of the wheel. What was the distance traveled by the light in this time interval?

A. $3R$ B. $4R$ C. $6R$ D. $8R$ E. NOTA

19. Find the radius of the largest circle that can fit inside the parabola $y = x^2$ while still touching the point (0,0).

(Hint: this is the radius of the osculating circle at (0,0).)

A. $\frac{1}{3}$ B. $\frac{1}{2}$ C. 1 D. 2 E. NOTA

20. Evaluate:

$$\lim_{x \to 0} \frac{3\tan^{-1}(x) - 3x + x^3}{3x^5}$$

A. 0 B. $\frac{1}{3}$ C. $\frac{1}{5}$ D. DNE E. NOTA

21. Consider the region enclosed by the petal of the polar rose $r = \sin(2\theta)$ that is completely in the first quadrant. In polar coordinates, what are the coordinates of the center of mass of this region?

A. $\left(\frac{128\sqrt{2}}{105\pi}, \frac{\pi}{4}\right)$ B. $\left(\frac{64\sqrt{2}}{105\pi}, \frac{\pi}{4}\right)$ C. $\left(\frac{128\sqrt{2}}{95\pi}, \frac{\pi}{4}\right)$ D. $\left(\frac{64\sqrt{2}}{95\pi}, \frac{\pi}{4}\right)$ E. NOTA

22. Find the interval of convergence of the following power series:

$$\sum_{n=1}^{\infty} \frac{n(2x-1)^n}{4^n}$$

A. $\left(-\frac{1}{2}, \frac{3}{2}\right)$ B. $\left[-\frac{5}{2}, \frac{3}{2}\right]$ C. $\left[-\frac{3}{2}, \frac{5}{2}\right]$ D. $\left[-\frac{3}{2}, \frac{5}{2}\right)$ E. NOTA

23. Which of the following series are convergent?

I. $\sum_{n=3}^{\infty} \frac{1}{n(\ln(n))^2}$ II. $\sum_{n=1}^{\infty} \frac{n+1}{\sqrt{n^3+n+1}}$ III. $\sum_{n=1}^{\infty} \frac{\sin^2(n) + e^{-n}}{n^2+1}$ IV. $\sum_{n=1}^{\infty} (-1)^n \left(\frac{n+1}{n}\right)^{n^2}$

A. I, III B. III, IV C. I, II, III D. I, II, III, IV E. NOTA

24. Let r be a real number satisfying $0 < r < 1$. Consider the curve defined by $y = \sqrt{r^2 - x^2}$, $r^2 \le x \le r$. What is the maximum area of the surface obtained by rotating the curve around the x −axis?

A. $\frac{4\pi}{27}$ B. $\frac{8\pi}{27}$ C. $\frac{4}{27}$ D. $\frac{40\pi}{27}$ E. NOTA

25. Evaluate:

$$\int_0^1 \left[\sum_{k=1}^{\infty} (-x^2)^k\right] dx$$

A. $\frac{\pi}{4} - 1$ B. $\frac{\pi}{4}$ C. $-\frac{\pi}{4} + 1$ D. $\frac{\pi}{2}$ E. NOTA

26. A particle moving in the xy-plane has the following position at time t:

$$\langle t^3 + t^2 + 1, 3t^2 + 1\rangle$$

At what time t is the particle stationary?

A. 0 B. 1 C. 2 D. 3 E. NOTA

27. Evaluate:

$$\int_0^1 (\ln(x))^4 dx$$

A. 6 B. 20 C. 24 D. 32 E. NOTA

28. What is the total length of the curve given in polar coordinates by $r = 2 + 2\cos(\theta)$?

A. 4 B. 8 C. 16 D. 32 E. NOTA

29. Evaluate:

$$\sum_{n=1}^{\infty} \ln\left(\frac{tan^{-1}(n)}{tan^{-1}(n+1)}\right)$$

A. $\ln\frac{\pi}{2}$ B. $\ln\frac{\pi}{4}$ C. $-\ln 4$ D. $-\ln 2$ E. NOTA

30. A submarine is built with a special viewing window, placed vertically in the side of the submarine, in the shape of a circle of radius 1 meter. The submarine dives so that the center of the window is at a depth of h meters, where $h > 1$. Calculate the total hydrostatic pressure exerted on the window, in terms of h. (The force, F, exerted by the fluid is $F = \rho g d A$, where ρ is the density of water: $1000 kg/m^3$, g is the acceleration due to gravity: $10 m/s^2$, d is the depth, and A is the area submerged.)

A. $5000\pi h$ B. $10000\pi h$ C. $20000\pi h$ D. $40000\pi h$ E. NOTA