For all questions, answer E) NOTA means none of the above answers is correct. Good luck and have fun!

1. The sum of 18 consecutive positive integers is a perfect square. What is the smallest value this sum could be?

   A) 169 B) 225 C) 289 D) 324 E) NOTA

2. There are six cups labeled 1 through 6 and six balls labeled 1 through 6. In how many ways can the balls be placed in the cups so that no ball is in the cup labeled with the same number, if each ball goes into one cup?

   A) 44 B) 63 C) 265 D) 620 E) NOTA

3. Each card in a deck has an integer written on it, and the integers on each of the 12 cards in the deck are consecutive. In a certain game, the number of points awarded for each turn is determined by drawing two cards and multiplying the numbers shown on the cards. If the points awarded in three turns are 40, 72, and 60, all of the following could be the smallest numbered card in the deck EXCEPT:

   A) -1 B) 0 C) 4 D) 6 E) NOTA

4. Suppose that $\sin a + \sin b = \sqrt{\frac{5}{3}}$ and $\cos a + \cos b = 1$. What is $\cos(a - b)$?

   A) ½ B) 1 C) $\frac{1}{3}$ D) $\frac{2}{3}$ E) NOTA

5. A bus travelling to the Hoover Dam overtakes a cyclist at 9:00 A.M. The bus reaches the Hoover Dam at 10:30 A.M. and after waiting for 1 hour, returns on the same path, passing the cyclist again at noon. Assuming that both the bus and the cyclist travel at constant speeds, when will the cyclist reach the Hoover Dam?

   A) 1:15 P.M. B) 1:30 P.M. C) 1:45 P.M. D) 2:00 P.M. E) NOTA

6. Rectangle ABCD is constructed in the coordinate plane parallel to the x- and y-axes. If the x- and y-coordinates of each of the vertices are integers that satisfy $3 \leq x \leq 11$ and $-5 \leq y \leq 5$, how many possible ways are there to construct rectangle ABCD?

   A) 396 B) 1260 C) 1980 D) 7920 E) NOTA

7. The sum of four consecutive odd numbers of increasing value is equal to the sum of three consecutive even numbers of increasing value. Given that the middle term of the even numbers is greater than 101 and less than 200, how many such sequences of even numbers can be formed?

A) 12 B) 17 C) 24 D) 33 E) NOTA

8. A box contains three pairs of blue gloves and two pairs of green gloves. Each pair consists of a left-hand glove and a right-hand glove. The gloves are separated and mixed in the box. If three gloves are randomly selected from the box, what is the probability that a matched set (a left- and right-hand glove of the same color) will be among the three gloves selected?

A) $\frac{3}{10}$ B) $\frac{23}{60}$ C) $\frac{7}{12}$ D) $\frac{41}{60}$ E) NOTA

9. A, B, C are digits in the range 0-9. Find the value of A+B+C if the sum of the three-digit number 3A4 and the four-digit number 1ACB equals the four-digit number 1C7C.

A) 6 B) 8 C) 13 D) 16 E) NOTA

10. What is the maximum value of $2\sin^6 x + 6\sin^4 x\cos^2 x + 6\sin^2 x\cos^4 x + 2\cos^6 x$?

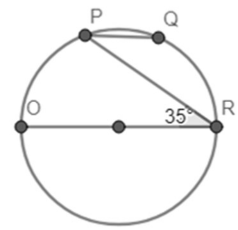
A) 8 B) 4 C) 2 D) $\sqrt{2}$ E) NOTA

11. Eric estimates that his crew of 5 men can finish building a swimming pool in 110 days if there is no rain, where the crew does not work on a rainy day and rain is the only factor preventing them from working. However, on the 61st day, after 5 days of rain, Eric hired 3 more people and finished the pool early. If the pool was done during the 100th day, how many days after day 60 had rain? Assume all workers work at the exact same rate, and there are no partial rainy/work days.

(A) 4 B) 5 C) 6 D) 7 E) NOTA

12. A sphere is inscribed in a cube with an edge length of 10. What is the shortest possible distance from one of the vertices of the cube to the surface of the sphere?

A) $5(\sqrt{2}-1)$
B) $10(\sqrt{2}-1)$
C) $5(\sqrt{3}-1)$
D) $10(\sqrt{3}-1)$
E) NOTA



13. In a circle with center C and diameter OR of length 18, a segment PQ is drawn parallel to OR. If m∠PRO = 35°, find the length of minor arc PQ.

A) 2π B) $\frac{9\pi}{4}$ C) $\frac{7\pi}{2}$ D) $\frac{9\pi}{2}$ E) NOTA

14. Soda A and Soda B can each be sold for a different profit percentage. If they are mixed in the ratio 1:2, the mixture can be sold for a 10% profit, and if they are mixed in the ratio 2:1, the mixture can be sold for a 20% profit. If the sodas are mixed in a 1:1 ratio, and the individual profit percentage on Soda A and Soda B is increased by 1/3 and 2/3 respectively, what profit will the mixture sell for? Assume the act of mixing the sodas does not change the profit percentage.

A) 18% B) 20% C) 21% D) 23% E) NOTA

15. A circus tent has one central pole 25 ft tall, and a dozen 20 ft poles arranged uniformly in a circle of diameter 24 ft around it. The tent material is taut between each of these points. Define the central angle of the tent as the angle of the tent material in the plane that cuts through two diametrically opposed tent poles. If the central angle of the tent measures θ, what is $\sin\theta$?

A) $\frac{120}{169}$ B) $\frac{5}{13}$ C) $\frac{12}{13}$ D) $\frac{24}{25}$ E) NOTA

16. A parking lot has 12 spaces in a row. Eight cars arrive, each of which requires one parking space, and their drivers choose spaces at random from among the available spaces. Buffy then arrives in his SUV, which requires 2 adjacent spaces. What is the probability that he is able to park?

A) $\frac{1}{3}$ B) $\frac{2}{3}$ C) $\frac{14}{55}$ D) $\frac{56}{165}$ E) NOTA

17. The sum of all the digits of the positive integer Q is equal to the three-digit number $X13$, where X is a single-digit positive integer. If $Q = 10^n - 49$, what is the value of n?

A) 24 B) 25 C) 26 D) 27 E) NOTA

18. Find the remainder when $\sum_{k=0}^{2019}\binom{k+2}{k}$ is divided by 2019.

A) 0 B) 1 C) 2 D) 674 E) NOTA

19. Let ABC be an isosceles triangle with AB = AC and m∠BAC = 20°. Point D is on side AC such that m∠CBD = 50°. Point E is on side AB such that m∠BCE = 60°. Find the measure of ∠CED.

A) 30 B) 15 C) 45 D) 25 E) NOTA

20. A geometric sequence is given by the expression: $g(n) = 5 * \left(-\frac{1}{2}\right)^n$. If the sequence begins with $n = 1$, what are the first two terms for which $|g(n) - g(n+1)| < \frac{1}{1000}$?

A) $g(10), g(11)$
B) $g(11), g(12)$
C) $g(12), g(13)$
D) $g(13), g(14)$
E) NOTA

21. Daniel is playing a game. He writes 3, 5, 7, 11, and 13 on five cards and distributes them among three envelopes, where they can be broken up in any way. He finds the product of all the cards in that envelope, which is now the "number" of the envelope. An envelope containing no cards has the number 1. He then puts the three envelope numbers in order from lowest to highest, and that is his "set". How many different sets can be produced by this process?

A) 41 B) 56 C) 89 D) 125 E) NOTA

22. Find the sum of all possible values of $x + y$ if $13x + 31y = 901$ and x and y are both positive integers.

A) 61 B) 104 C) 129 D) 183 E) NOTA

23. Evaluate the sum $\sum_{n=0}^{\infty} 2\cos^n(2\theta)$ for $0 < \theta < \pi/2$.

A) $cot\ \theta$ B) $\csc\theta\ sec\ \theta$ C) $\sec^2\theta$ D) $csc^2\theta$ E) NOTA

24. A function f is defined by $f(z) = (4 + i)z^2 + \mu z + \gamma$ for all complex numbers z, where μ and γ are complex numbers and $i^2 = -1$. Suppose that $f(1)$ and $f(i)$ are both real. What is the smallest possible value of |μ| + |γ|?

A) 0 B) $\sqrt{2}$ C) $2\sqrt{2}$ D) 4 E) NOTA

25. A hundred identical unit cubes are arranged in four blocks: a single cubic block, a 2x2x2 block, a 3x3x3 block, and a 4x4x4 block. The surfaces of these blocks are then painted, and inward faces are not painted. These four blocks are then dismantled and reassembled as a 10x10x1 block. The top and the four sides of this 10x10x1 block must be painted, but there is no requirement for paint on the bottom. What is the minimum number of individual faces that must be painted to accomplish this?

A) 0 B) 5 C) 9 D) 16 E) NOTA

26. How many positive integers have exactly three proper divisors that are all less than 40? (A proper divisor of positive integer n is a positive integral divisor of n that is not n itself.)

A) 66 B) 69 C) 72 D) 85 E) NOTA

27. The value of $\sec\frac{\pi}{12}$ can be expressed in the form $a\sqrt{\sqrt{b}+c\sqrt{d}}$, where a, b, c, d are integers, a is even, and d is square-free. What is the product $abcd$?

A) 16 B) –12 C) –24 D) –36 E) NOTA

28. Let a, b, and c be real numbers such that $a - 7b + 8c = 4$ and $8a + 4b - c = 7$. What is $a^2 - b^2 + c^2$?

A) 1 B) 4 C) 7 D) 8 E) NOTA

29. Jenny and Konwoo were tied after the first round of a four-round math competition. The number of points Jenny scored in each of the four rounds formed an increasing geometric sequence of whole numbers, while the number of points scored by Konwoo in each of the four rounds formed an increasing arithmetic sequence of whole numbers. At the end of the fourth round, Jenny had won by one point. Neither scored more than 100 points. What was the total number of points scored by the two in the first two rounds?

A) 30 B) 31 C) 32 D) 33 E) NOTA

30. In how many ways can 1 black knight and 1 white knight be placed on an 8x8 chessboard so that they cannot mutually attack each other? Attacking means that each can reach the other in one move. Knights can move either two squares horizontally and one square vertically, or two squares vertically and one square horizontally.

A) 3696 B) 2864 C) 3948 D) 3990 E) NOTA