1. **B.** $x + (x + 1) + (x + 2) + \cdots + (x + 17) = 18x + (1 + 2 + \cdots + 17) = 18x + 17(18)2 = 18x + 153$. Factoring this sum, we get $9(2x + 17)$. In order for this product to be a perfect square, $2x + 17$ must be a perfect square since 9 is already a perfect square. The smallest x which accomplishes this is 4 since $2(4) + 17 = 25$. So the smallest value the sum could be is $9(25) = 225$.

2. **C.** The total number of ways the 6 balls can be put into the 6 cups is 6!. We must subtract the arrangements where one ball is placed in the correct cup (the number of ways we can choose the ball that is to be put in the correct cup is ${}_6C_1$ and the other balls ban be placed in 5! ways, making a total of ${}_6C_3 1 * 5!$ ways to accomplish this). Then we must add back the arrangements where 2 balls are placed in the correct cup, since we subtracted out this case twice previously (number of ways to choose the 2 correct balls is ${}_6C_2$, leaving 4! ways to arrange the others and ${}_6C_2 * 4!$ ways total). Following similar logic, the arrangements where 3 balls are placed correctly are included twice so we must subtract those (${}_6C_3 * 3!$). If we follow this pattern, the number of ways to place the 6 balls in the wrong cups is 6! - ${}_6C_1$*5! + ${}_6C_2$*4! - ${}_6C_3$*3! + ${}_6C_4$*2! - ${}_6C_5$*1! + ${}_6C_6$*0! = 265.

3. **D.** For 40 to be a valid score, the deck must include a pair of numbers that multiply to 40 and are within 12 integers apart. By checking the factors of 40, we can see that $4x10$ and $5x8$ are the only such pairs satisfying these conditions, so either 4 or 5 must be a card. 6 cannot be the smallest card.

4. **C.** Sum the squares of the two equations: $\sin^2 a + \sin^2 b + 2 \sin a \sin b + \cos^2 a + \cos^2 b + 2 \cos a \cos b = \frac{5}{3} + 1 \rightarrow 2 + 2 \sin a \sin b + 2 \cos a \cos b = \frac{8}{3} \rightarrow 2(\cos a \cos b + \sin a \sin b) = \frac{2}{3}$. This is the cosine difference identity so $\cos(a - b) = \frac{1}{3}$.

5. **B.** Let the distance from the cyclist to Hoover Dam be $3x$. It takes the bus $\frac{3}{2}$ hours to travel this distance, so the rate of the bus is $2x$/hour. The bus meets the cyclist again after ½ hours, so the distance covered by the bus = $rate * time = 2 * ½ = x$. At this point, the cyclist has travelled a distance of $3x - x = 2x$. Therefore, speed of cyclist= $\frac{2}{3}x$/hour. The cyclist is still x away from Hoover Dam. $T = \frac{d}{r} = \frac{x}{\frac{2}{3}x} = 3/2$ hours. 12+ 1.5 = 1:30 PM.

6. **C.** A rectangle can be made by the intersection of any two vertical lines and any two horizontal lines on the x domain [3,11] and y domain [-5,5], so ${}_9C_2 * {}_{11}C_2 = 1980$.

7. **A**. Sum of four consecutive odd numbers:

$$(2a - 3) + (2a - 1) + (2a + 1) + (2a + 3) = 8a$$

Sum of three consecutive even numbers:

$$(2b - 2) + 2b + (2b + 2) = 6b$$

Given that $8a = 6b$ or $\frac{a}{b} = \frac{3}{4}$, a and b can be any integers. So, a must be a multiple of 3

and b must be a multiple of 4. Since $101 < 2b < 200$ → $51 \leq b < 100$.
b also has to be a multiple of 4, so the values that b can take are
$52, 56, 60 \ldots 92, 96.$ $\frac{96-52}{4} + 1 = 12.$

8. **D.** Let us calculate the probability of not getting a matched pair. This occurs if we pick three same hand blue gloves, two same hand blue gloves and any green glove, or two same hand green gloves and any blue glove.
BBB: $\frac{6}{10} * \frac{2}{9} * \frac{1}{8} = \frac{1}{60}$
BBG: $\left(\frac{6}{10} * \frac{2}{9} * \frac{4}{8}\right) * 3 = \frac{12}{60}$ (multiplying by 3 for the 3 permutations of BBG, BGB, GBB)
GGB: $(\frac{4}{10} * \frac{1}{9} * \frac{6}{8}) * 3 = \frac{6}{60}$
$P = 1 - (\frac{1}{60} + \frac{12}{60} + \frac{6}{60}) = \frac{41}{60}$.

9. **B.** Looking at the hundreds' place, either 3+A=C or 4+A=C (if carrying was necessary from the tens place). Assuming the latter, either A+C=17 or A+C=16. However, solving 4+A=C with either of these gives non-integer solutions (first equation) or C =10 (second equation), neither of which are possible. Therefore, 3+A=C is true, making A+C=7 or A+C=6 (if carrying was necessary from the ones place). However, solving 3+A=C with the second of these equations gives non-integer solutions. Therefore, 3+A=C and A+C=7 and solving gives A=2 and C=5. Since no carrying was necessary from the ones' place, 4+B=C=5 and B=1. A+B+C=2+1+5=8.

10. **C.** Using the Binomial Expansion Theorem to write the expression as $2(sin^2x + cos^2x)^3 = 2 * 1^3 = 2$. Since this expression evaluates to a constant, the maximum value is 2.

11. **B**. It rained on 5 days of the first 60 days, meaning the crew has worked for 55 days and has $110 - 55 = 55$ more days of work. Once 3 more people were hired, the speed of the construction increased to $\frac{8}{5}$, and the days needed to finish is now $55/\,(\frac{8}{5}) = 34.375$ days (rounding up for partial days). They finished the second half in $100 - 60 = 40$ days, so it rained for $40 - 35 = 5$ days.

12. **C.** The shortest distance from a vertex to the cube to the surface of the sphere would be the diagonal of the cube D minus the diameter ($2s$) of the sphere divided by 2.

$$D = \sqrt{2s^2 + s^2} = \sqrt{2 * 100 + 100} = \sqrt{300}$$

Shortest Distance = $\frac{\sqrt{300}-10}{2} = 5(\sqrt{3} - 1)$.

13. **A.** The Central Angle Theorem states that the measure of inscribed angle is always half the measure of the central angle. Therefore, $\angle PCO = 2\angle PRO = 70$. Since PQ is parallel to OR,

$\angle QPR = \angle PRO = 35$ and $\angle QCR = 2\angle QPR = 70$. $\angle PCQ = 180 - (\angle PCO + \angle QCR) = 180 - 70 - 70 = 40$. Minor arc PQ $= \frac{40}{360} * circumference = \frac{2\pi}{9} = 2\pi$.

14. B. Call the profit on Soda A x% and the profit on Soda B y%.

When they are mixed in the ratio 1:2 (total of 3 parts), the profit is 10%: $\frac{x + 2y}{3} = 10$.

When they are mixed in the ratio 2:1 (total of 3 parts), the profit is 20%: $\frac{2x + y}{3} = 20$.
Solving gives: $x = 30$% and $y = 0$%. After the individual profit percentage on them are increased by 4/3 and 5/3 times respectively, the profit becomes 40% and 0% respectively. A mixture of 1:1 will sell for a profit of $\frac{40 + 0}{2} = 20\%$.

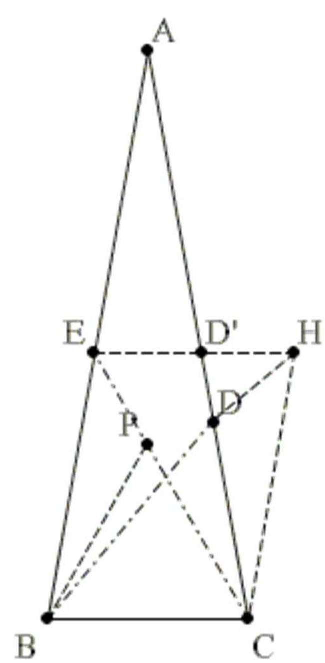
15. A. Consider the right triangle with one vertex at the top of the center pole, one vertex at the top of a support pole, and one vertex 20 ft up on the center pole. This is a 5-12-13 triangle. The interior angle of the tent is double an angle φ that is opposite the 12 ft side of this triangle. $sin\ \theta = \sin 2\varphi = 2(\sin\varphi)(\cos\varphi) = 2\left(\frac{5}{13}\right)\left(\frac{12}{13}\right) = \frac{120}{169}$.

16. E. Let us label the vacant spaces V and the occupied spaces O. The number of ways that the cars can be parked in the lot is the permutation of 8 V's and 4 O's, or $\frac{12!}{8!4!} = 495$. Buffy will not be able to park if no two V's are adjacent, which occurs when you insert the 4 V's into the 9 spaces between the O's, or $\frac{9!}{5!4!}$ 126. The number of configurations that Buffy is able to park is $495 - 126 = 369$, and the probability he can park is $\frac{369}{495} = \frac{41}{55}$.

17. B. 10^n has $n + 1$ digits (1 and n zeros). $10^n - 49$ will have n digits ($n - 2$ 9's and a 51 at the end). The sum of all the digits of $10^n - 49$ is equal to the three-digit number $X13$ → $9(n-2) + 5 + 1 = X13$ → $9n - 12 = X13$ → $9n = X25$. $X25$ is divisible by 9 → the sum of its digits must be divisible by 9 → $X = 2$ → $9n = 225$ → $n = 25$.

18. D. Notably, $\binom{k+2}{k} = \binom{k+2}{2}$, so by the Hockey Stick Identity, $\sum_{k=0}^{2019}\binom{k+2}{2} = \sum_{k=2}^{2021}\binom{k}{2} = \binom{2022}{3} = \frac{2022*2021*2020}{6} = 337 * 2021 * 2020$. Taking this mod 2019 gives $337 * 2 * 1 = 674$.

19. A. Draw a line through E parallel to BC and another through C parallel to AB. The two intersect at a point H forming a parallelogram BCHE. Let P be the point on CE that completes the equilateral triangle BCP. Then BP = BC = CD and BE = CH. ∠EBP = 20° and ∠HCD = 20°; by SAS, ΔEBP = ΔHCD. ∠CHD = ∠BEC = 40°. Since ∠CHE = 80°, HD is the bisector of angle CHE. On the other hand, CD is the bisector of angle



ECH. The point D is then the intersection of ΔECH's angle bisectors (incenter). DE is the bisector of angle CEH and ∠CED = 30°.

20. D. $|g(n) - g(n+1)| = |5 * \left(-\frac{1}{2}\right)^n - 5 * \left(-\frac{1}{2}\right)^{n+1}|$. Factoring out a $5 * \left(-\frac{1}{2}\right)^n$:
$|5 * \left(-\frac{1}{2}\right)^n - 5 * \left(-\frac{1}{2}\right)^{n+1}| = |5 * \left(-\frac{1}{2}\right)^n * (1 - \left(-\frac{1}{2}\right)^1)| = |5 * \left(-\frac{1}{2}\right)^n * \frac{3}{2}| = \frac{15}{2} * \left(\frac{1}{2}\right)^n$.
We need to find the least value of n for which $\frac{15}{2} * \left(\frac{1}{2}\right)^n < \frac{1}{1000}$ → $2^n > 7500$. The least n for which this inequality is true is 13, so the first two terms are $g(13)$ and $g(14)$.

21. A. Case 1: 1 used envelope → 1 way
Case 2: 2 used envelopes → broken up into 4-1 (${}_5C_4$) or 3-2 (${}_5C_3$) = 5+10 = 15 ways
Case 3: All envelopes used → broken up into 3-1-1 (${}_5C_3$) or 2-2-1 (${}_5C_2 * {}_3C_2/2$), dividing by two because the two pairs are double-counted = 10+15 = 25 ways

Total is 1+15+25 = 41 ways.

22. B. Taking the entire equation modulo 13 to get $5y \equiv 4 \; mod \; 13$. By inspection, $y \equiv 6$. This yields two solutions, (55, 6) and (24, 19). $55 + 6 + 24 + 19 = 104$.

23. D. The series is an infinite geometric one, so the sum can be found using the formula $\frac{a_1}{1-r} = \frac{2}{1-\cos 2\theta} = \frac{2}{1-(1-2\sin^2\theta)} = \frac{2}{2\sin^2\theta} = \csc^2\theta$.

24. B. μ and γ are complex numbers, meaning that $\mu = a + bi$ and $\gamma = c + di$ for some real numbers a, b, c, d. Expanding out $f(1)$ and $f(i)$ gives $f(1) = 4 + a + c + i(1 + b + d)$ and $f(i) = -4 - b + c + i(-1 + a + d)$. Both of these are real, meaning that the imaginary parts equal zero, so $1 + b + d = 0$ and $-1 + a + d = 0$. We are looking for the smallest sum of $\sqrt{a^2 + b^2} + \sqrt{c^2 + d^2}$. Substituting the two equations in gives $\sqrt{(1-d)^2 + (-1-d)^2} + \sqrt{c^2 + d^2} = \sqrt{2 + 2d^2} + \sqrt{c^2 + d^2}$. This is minimized when c=0 and d=0, giving $\sqrt{2+0} + \sqrt{0+0} = \sqrt{2}$ as the answer.

25. C. We can determine the number of faces to be painted by counting the small cubes that are not painted because there are more than enough cubes painted on 3 sides to cover the 4 corners of the 10x10x1 prism and enough cubes painted on 2 or 3 sides to cover the edges.
In a single cubic box - no such cubes
In a 2*2*2 box - also no such cubes
In a 3*3*3 box - 1 cube in the center of the box
In a 4*4*4 box - 8 cubes in the 2x2x2 section of the center of the box
As a we have to use all 100 small cubes to make the 10x10x1 box, $8 + 1 = 9$ faces of 9 not painted cubes have to be painted.

26. B. If n is one of these integers, one of its factors must be one, so n must be in the form $n = p * q$ or $n = p^3$ for distinct prime numbers p and q. For the first case, the three proper divisors are $1, p$, and q. We need to pick two prime numbers less than 40, of which there are 12 $(2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37)$, and that gives ${}_{12}C_2 = 66$ numbers of the first type. For the second case, the three proper divisors are $1, p$, and p^2. We need to pick a prime number whose square is less than 40, of which there are 3 $(2, 3, 5)$, and that gives 3 numbers of the second type. Therefore, there are $66 + 3 = 69$ integers satisfying the conditions.

27. C. $\cos\frac{\pi}{12} = \cos\frac{\frac{\pi}{6}}{2} = \sqrt{\frac{1+\cos\frac{\pi}{6}}{2}} = \frac{1}{2}\sqrt{\frac{1}{2} + \frac{\sqrt{3}}{4}} = \frac{1}{2}\sqrt{2 + \sqrt{3}} \rightarrow \sec\frac{\pi}{12} = \frac{1}{\cos\frac{\pi}{12}} = \frac{2}{\sqrt{2+\sqrt{3}}} = 2\sqrt{2 - \sqrt{3}} \rightarrow a = 2, c = -1, b = 4, d = 3 \rightarrow abcd = -24.$

28. A. By rearranging the equations, we get $a + 8c = 7b + 4$ and $8a - c = 7 - 4b$. Squaring both, we get $a^2 + 16ac + 64c^2 = 49b^2 + 56b + 16$ and $64a^2 - 16ac + c^2 = 16b^2 - 56b + 49$. Adding the equations and dividing by 65 gives $a^2 + c^2 = b^2 + 1$, so $a^2 - b^2 + c^2 = 1$.

29. E. Let us write Jenny and Konwoo's scores as (a, an, an^2, an^3) and $(a, a + m, a + 2m, a + 3m)$. Konwoo's sum, $4a + 6m$, is even. Therefore, since Konwoo's sum is one less than Jenny, Jenny's score should be odd. However, if all of (a, an, an^2, an^3) have the same parity, the sum will be even. If a is even, then the rest of the scores will be even, so a is odd. → n is even. If $n = 4$, only $a = 1$ works given that the total score of each is less than 100, and this combination doesn't work. Therefore, $n = 2$, and trying out $a = (1, 3, 5)$, only $a = 5$ works. The scores are $(5, 10, 20, 40)$ and $(5, 14, 23, 32)$, and the answer is $5 + 10 + 5 + 14 = 34$.

30. A. There are 64 places to place one knight and 63 places to place the other, giving 4032 total positions the knights can be in. We subtract the positions where the knights can attack each other, which happens if they are at opposite corners of a 3×2 or 2×3 rectangle. A 3×2 rectangle can be determined by its lower left corner square, which must be on one of the first 6 columns and one of the first 7 rows, so the number of such rectangles is $6 \times 7 = 42$. The number of 2×3 rectangles is the same, so the total number of rectangles is $2 \times 42 = 84$. For a pair of mutually attacking knights, the white knight must be at one of the 4 corners of one of the 84 rectangles, and the black knight at the opposite corner, so the number of positions for the two knights is $84 \times 4 = 336$. $4032 - 336 = 3696$.