For all questions, answer choice "E) NOTA" means that none of the above answers is correct.

1. On their school break, Ksenia and Faustine worked for a farmer by picking fruit. Ksenia worked for 5 days and Faustine worked for 3 days. The farmer paid them \$1000, which they shared in the same ratio as the days they worked. Faustine's share was

A) \$250 B) \$300 C) \$325 D) \$375 E) NOTA

2. How many permutations of the letters in the word "batmen" end in a vowel?

A) 120 B) 240 C) 480 D) 720 E) NOTA

3. Anika measures three sides of a rectangle and gets a total of 80 inches. Heeyoon measures three sides of the same rectangle and gets a total of 88 inches. What is the perimeter of the rectangle in inches?

A) 96 B) 112 C) 132 D) 156 E) NOTA

4. The difference between two numbers is 20. When 4 is added to each number, the larger is three times the smaller. What is the larger of the two original numbers?

A) 26 B) 30 C) 34 D) 38 E) NOTA

5. Three different dinosaur fossils have been found in the kingdom of Mathematicus. Cantoraptor is twice as old as Kovalevskasaurus, though when Kovalevskasaurus was alive, Cantoraptor was twice as old as Fermatdactyl. The sum of the ages of the three fossils is 360 million years. How many million years after Fermatdactyl did Kovalevskasaurus live? Assume the age of each dinosaur is negligible compared to the age of the fossils.

A) 20 B) 40 C) 60 D) 80 E) NOTA

6. Simplify the following expression to $a + bi$ form.

$$\frac{i^2 + 4i}{7i + 2}$$

A) $\frac{15}{53} - \frac{26}{53}i$ B) $\frac{26}{53} - \frac{15}{53}i$ C) $\frac{30}{53} - \frac{1}{53}i$ D) $\frac{30}{53} + \frac{1}{53}i$ E) NOTA

7. Find the area of the triangle formed by the vectors $\langle 4, -1, 1\rangle$ and $\langle -2, 1, 3\rangle$.

A) $3\sqrt{6}$ B) 6 C) $6\sqrt{6}$ D) $6\sqrt{7}$ E) NOTA

8. After a genetics experiment, the numbers of pea plants having certain characteristics were tallied as follows.

- Were tall—22
- Had green peas—25
- Had smooth peas—39
- Were tall and had green peas—9
- Were tall and had smooth peas—17
- Had green peas and smooth peas—20
- Were tall and had green peas and had smooth peas—6

How many plants were not tall but had peas which were smooth and green?

A) 14 B) 18 C) 20 D) 28 E) NOTA

9. Compute the value of k so that the coordinates of the solution to the system of equations below have a sum of 5.

$$\begin{cases} x - y + z = 0 \\ x + y - z = 4 \\ x + 2y + 3z = k \end{cases}$$

A) 8 B) 8.5 C) 9 D) 9.5 E) NOTA

10. Compute the limit below.

$$\lim_{n\to\infty} \frac{1^2 + 2^2 + 3^2 + \cdots + n^2}{n^3}$$

A) 0 B) $\frac{1}{6}$ C) $\frac{1}{3}$ D) $\frac{1}{2}$ E) NOTA

11. For real values of t, a particle has position in the xy-plane given by the parametric equations $x_1(t) = 2t^2 - t + 3$ and $y_1(t) = t^3 + 1$. A second particle has position in the xy-plane given by $x_2(t) = t^3 - 2t^2 + 9$ and $y_2(t) = t^2 + 9t - 8$. At what point (x, y) do the particles collide?

A) $(2, 3)$

B) $(9, -26)$

C) $(18, 28)$

D) The particles do not collide for any value of t.

E) NOTA

12. Given an integer $N > 1$, we may compute the sum of N and its second largest factor. For example, with $N = 55$, the sum is $55 + 11 = 66$. For how many integers is this sum equal to 48?

A) 0 B) 1 C) 2 D) 3 E) NOTA

13. Compute the volume of the solid formed by revolving the region between the curves $y = 0$, $x = 0$, $x = \ln(2)$, and $y = e^x$ about the line $y = -1$.

A) $\pi(\ln(4) - 1)$ B) $\frac{7\pi}{2}$ C) $2\pi \ln(2)(\ln(2) + 1)$ D) $\frac{9\pi}{2}$ E) NOTA

14. Let $f(k) = 1^k + 2^k + 3^k + 4^k + 5^k + 6^k + 7^k + 8^k + 9^k$, where k is a nonnegative integer. How many possible values of the units digit of $f(k)$ are there?

A) 1 B) 2 C) 3 D) 4 E) NOTA

15. A production machine is strangely erratic. When you push its button, one of three things happens.

- 60% of the time it goes into mode A and is done with its production in 9 minutes.
- 30% of the time it goes into mode B, beeps uselessly for 5 minutes, and produces nothing, so that you have to try again.
- 10% of the time it goes into mode C, buzzes uselessly for 3 minutes, and produces nothing, so that you have to try again.

Find the expected time, in minutes, to get the job done.

A) 10 B) 10.5 C) 11 D) 12 E) NOTA

16. Positive integers x and y satisfy $(2^x + 1)(2^y - 1) = 131327$. Compute the value of $x + y$.

A) 13 B) 14 C) 15 D) 16 E) NOTA

17. For what values of k does the equation $2^x k + 2^{-x} = 3$ have a single real root?

A) $k = 0$ and $k = \frac{9}{4}$ B) $0 \le k < \frac{9}{4}$ C) $k < \frac{9}{4}$ D) $k \le 0$ and $k = \frac{9}{4}$ E) NOTA

18. A solid ball 10 inches in diameter is placed within a cubic box 10 inches on each side. Approximately what percent of the box is filled with air, rounded to the nearest whole percent?

A) 47% B) 48% C) 49% D) 50% E) NOTA

19. Points $A(-2, 9)$ and $B(4, 3)$ lie on the graph of the parabola $y = x^2 - 3x - 1$. There is a point C with coordinates (m, n), for $-2 < m < 4$, such that C also lies on the parabola and maximizes the area of triangle ABC. Compute $m + n$.

A) $-\frac{7}{4}$ B) -2 C) 0 D) 7 E) NOTA

20. Find the area enclosed by the graph of $(x^2 + y^2 - x)^2 = x^2 + y^2$.

A) $\frac{\pi}{2}$ B) $\frac{3\pi}{2}$ C) $\frac{5\pi}{2}$ D) $4\pi^2 - 2\pi$ E) NOTA

21. A student noticed that in a list of five integers, the mean, median, and mode were consecutive integers in ascending order. What is the largest possible range for these five integers?

A) 5 B) 6 C) 7 D) 8 E) NOTA

22. A particle moves along the x-axis so that at any time t, its position is given by $x(t) = \frac{1}{2}\sin t + \cos 2t$. What is the acceleration of the particle at t = $\frac{\pi}{2}$?

A) 0.5 B) 1.5 C) 2.5 D) 3.5 E) NOTA

23. $\lim_{x\to\infty} \frac{x^3 - 2x^2 + 3x - 4}{4x^3 - 3x^2 + 2x - 1}$ =?

A) -1 B) 0 C) 0.25 D) 1 E) NOTA

24. Suppose x, y, and z are nonnegative integers. Compute the number of solutions to the equation $2x + y + z = 28$.

A) 175 B) 200 C) 351 D) 378 E) NOTA

25. Najifa spent a few hours writing the positive integers from 1 to 2019 on the board. She added up all her numbers and divided by 2019. She was very surprised that the result was an integer! Thinking that something must be wrong, she looked over her list, and she found out that she accidentally never wrote down three consecutive integers (so she wrote down only 2016 integers, not 2019 integers). The sum of the digits of those three integers is p. Find the largest possibility for p.

A) 39 B) 42 C) 45 D) 48 E) NOTA

26. We define *splendiferous* numbers in the following way: every one-digit prime is splendiferous, and a prime number of at least two digits is splendiferous if and only if the two numbers obtained from it by omitting either its first or last digit are also splendiferous. Compute the sum of all splendiferous numbers.

A) 77 B) 130 C) 336 D) 576 E) NOTA

27. $\int_2^4 |x-3|\,dx=?$

A) -2 B) -1 C) 0 D) 1 E) NOTA

28. To *concatenate* two integers a and b means to append a to b to form a new integer; for example, the concatenation of 38 and 491 is 38491. Suppose two prime numbers p and q have a difference of 100, and the concatenation of the larger and the smaller is another prime number. Compute the sum of the digits of p and q.

A) 7 B) 9 C) 15 D) 21 E) NOTA

29. Compute $2 + 2 \cdot 2^2 + 3 \cdot 2^3 + 4 \cdot 2^4 + 5 \cdot 2^5 + \cdots + 200 \cdot 2^{200}$.

A) $199 \cdot 2^{201} - 2$ B) $199 \cdot 2^{201} - 1$ C) $199 \cdot 2^{201} + 1$ D) $199 \cdot 2^{201} + 2$ E) NOTA

30. The set S consists of distinct integers such that the smallest is 0 and the largest is 2079. What is the minimum possible average value of the numbers in S?

A) 61 B) 62 C) 63 D) 64 E) NOTA