#0 Mu Bowl
MAΘ National Convention 2019

Let

$$A = \lim_{x\to 0}\left(\frac{x - x^3}{x^2 + 2x}\right) \qquad B = \lim_{x\to\infty}\left(1 - \frac{5}{x}\right)^{3x}$$

$$C = \lim_{x\to\infty}\left(\frac{3x^2 + x - 1}{2x^2 - x^3 + 7}\right) \qquad D = \lim_{x\to\infty}\left(\sqrt{x^2 + 2x} - \sqrt{x^2 - x}\right)$$

Find $\frac{AD}{B+C}$.

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Solution:

$A = \lim_{x\to 0}\left(\frac{x-x^3}{x^2+2x}\right) = \lim_{x\to 0}\left(\frac{x(1-x^2)}{x(x+2)}\right) = \frac{1}{2}$ $\quad B = \lim_{x\to\infty}\left(1 - \frac{5}{x}\right)^{3x} = e^{-15}$

$C = \lim_{x\to\infty}\left(\frac{3x^2+x-1}{2x^2-x^3+7}\right) = 0$ $\quad D = \lim_{x\to\infty}\left(\sqrt{x^2 + 2x} - \sqrt{x^2 - x}\right) = \lim_{x\to\infty}\left(\frac{\sqrt{x^2+2x}+\sqrt{x^2-x}}{(x^2+2x)-(x^2-x)}\right) = \frac{2}{3}$

$\frac{AD}{B+C} = \frac{\left(\frac{1}{2}\right)\left(\frac{2}{3}\right)}{0+e^{-15}} = \frac{e^{15}}{3}$.

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Let

$$A = \frac{d}{dx}[e^{-x}\cos(x^2)]|_{x=0} \qquad B = \frac{d}{dx}[x(x^2+1)^2(x^3+2)^3]|_{x=1}$$

$$C = \frac{d}{dx}\left[(x+1)^{x^3-1}\right]\Big|_{x=1} \qquad D = \frac{d}{dx}\left[\frac{\tan^2(x)}{x+\sin(x)+1}\right]\Bigg|_{x=0}$$

Find $A + B + C + D$.

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Solution:

$A = \frac{d}{dx}[e^{-x}\cos(x^2)]|_{x=0} = [e^{-x}(-2x\sin(x^2)) - e^{-x}\cos(x^2)]_{x=0} = -1$

B: $y = x(x^2+1)^2(x^3+2)^3 \rightarrow \ln(y) = \ln(x) + 2\ln(x^2+1) + 3\ln(x^3+2) \rightarrow y' = y * \left(\frac{1}{x} + \frac{4x}{x^2+1} + \frac{9x^2}{x^3+2}\right) \rightarrow y'(1) = y(1) * \left(1 + \frac{4}{2} + \frac{9}{3}\right) = 1 * 2^2 * 3^3 * (1+2+3) = 648$

C: $y = (x+1)^{x^3-1} \rightarrow \ln(y) = (x^3-1)\ln(x+1) \rightarrow y' = y * \left(\frac{x^3-1}{x+1} + 3x^2\ln(x+1)\right) \rightarrow y'(1) = 1 * (0 + 3\ln(2)) = 3\ln(2)$.

$D = \frac{d}{dx}\left[\frac{\tan^2(x)}{x+\sin(x)+1}\right]\Big|_{x=0} = \left[\frac{(x+\sin(x)+1)(2\,\mathrm{ta}\;\,(x)\sec^2(x)) - \tan^2(x)(1+\cos(x))}{(x+\sin(x)+1)^2}\right]_{x=0} = 0$.

$A + B + C + D = 647 + 3\ln(2)$.

Let

$$f(x) = \frac{1-x^2}{a}$$

$$g(x) = a(x^2 - 1)$$

for a positive real constant a.

Find the minimum possible area of the finite region bounded by $f(x)$ and $g(x)$.

Solution:

The area is $2\int_0^1 \frac{1-x^2}{a} - a(x^2-1)\,dx = 2\int_0^1 \left(a+\frac{1}{a}\right) - \left(a+\frac{1}{a}\right)x^2\,dx = 2\left(a+\frac{1}{a}\right)\left[x - \frac{1}{3}x^3\right]_0^1 = \frac{4}{3}\left(a+\frac{1}{a}\right)$. So then $R(a) = a + \frac{1}{a} \rightarrow R' = 1 - \frac{1}{a^2} = 0 \rightarrow a = 1 \rightarrow$ the minimum area is $\frac{8}{3}$.

Let

$$A = \lim_{n\to\infty} \sum_{k=1}^{n} \frac{1}{k+n} \qquad B = \lim_{n\to\infty} \prod_{k=1}^{n} \left(1+\frac{k}{n}\right)^{\frac{2}{n}}$$

$$C = \lim_{x\to 0} \frac{4\arctan(1+x)-\pi}{x} \qquad D = \lim_{x\to 0} \frac{\arctan(x)-x}{x^3}$$

Find $A+\ln(B)+C+D$.

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Solution:

$A = \lim\limits_{n\to\infty} \sum_{k=1}^{n} \frac{1}{k+n} = \lim\limits_{n\to\infty} \sum_{k=1}^{n} \frac{1}{n}\frac{1}{\frac{k}{n}+1} = \int_1^2 \frac{1}{x}\,dx = \ln(2).$

$B = \lim\limits_{n\to\infty} \prod_{k=1}^{n} \left(1+\frac{k}{n}\right)^{\frac{2}{n}} \to \ln(B) = 2\lim\limits_{n\to\infty} \sum_{k=1}^{n} \frac{1}{n}\ln\left(1+\frac{k}{n}\right) = 2\int_1^2 \ln(x)\,dx = 2[x\ln(x)-x]_1^2 =$
$4\ln(2)-2.$

$C = \lim\limits_{x\to 0} \frac{4\arctan(1+x)-\pi}{x} = 4\lim\limits_{x\to 0} \frac{\arctan(1+x)-\frac{\pi}{4}}{x} = 4\frac{d}{dx}[\arctan(t)]_{t=1} = \frac{4}{2} = 2.$

$D = \lim\limits_{x\to 0} \frac{\arctan(x)-x}{x^3} = \lim\limits_{x\to 0} \frac{\left(x-\frac{x^3}{3}+\frac{x^5}{5}-\cdots\right)-x}{x^3} = -\frac{1}{3}.$

So $A+\ln(B)+C+D = 5\ln(2)-\frac{1}{3}$.

Let

$$f(x) = x^{x+x^{x+x^{x+x^{\cdot^{\cdot^{\cdot}}}}}}$$

$$g(x) = \sqrt{4x^2 + \sqrt{4x^2 + \sqrt{4x^2 + \cdots}}}$$

If

$$A = \frac{d}{dx}[f(x)]_{x=1} \qquad B = \frac{d}{dx}[g(x)]_{x=1}$$

$$C = \int_1^2 (\ln(f(x)) - f(x)\ln(x))\,dx \qquad D = \int_0^1 x\,g(x)\,dx$$

Find $A + 17B + 4C + 96D$.

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Solution:

$f(x) = x^{x+x^{x+x^{x+x^{\cdot^{\cdot^{\cdot}}}}}} \to f = x^{x+f} \to \ln(f) = (x+f)\ln(x) \to \ln(f) - f\ln(x) = x\ln(x)$. Therefore $\frac{f'}{f} - f'\ln(x) - \frac{f}{x} = \ln(x) + 1 \to f'(x) = \frac{\ln(x)+1+\frac{f}{x}}{\frac{1}{f} - \ln(x)} \to f'(1) = A = \frac{0+1+f(1)}{\frac{1}{f(1)} - 0} = 2$. Further, from above,

$$C = \int_1^2 (\ln(f) - f\ln(x))\,dx = \int_1^2 x\ln(x)\,dx = \left[\frac{1}{2}x^2\ln(x)\right]_1^2 - \int_1^2 \frac{1}{2}x\,dx = 2\ln(2) - \left[\frac{1}{4}x^2\right]_1^2 = 2\ln(2) - \frac{3}{4}$$

$g(x) = \sqrt{4x^2 + \sqrt{4x^2 + \sqrt{4x^2 + \cdots}}} \to g^2 = 4x^2 + g \to g^2 - g - 4x^2 = 0 \to g = \frac{1+\sqrt{1+16x^2}}{2} \to g' = \frac{8x}{\sqrt{1+16x^2}} \to g'(1) = B = \frac{8}{\sqrt{17}}$. Also $D = \int_0^1 x\,g(x)\,dx = \int_0^1 \frac{1}{2}x + \frac{x}{2}\sqrt{1+16x^2}\,dx = \left[\frac{1}{4}x^2 + \frac{1}{64}\frac{2}{3}(1+16x^2)^{\frac{3}{2}}\right]_0^1 = \frac{1}{4} + \frac{17\sqrt{17}}{96} - \frac{1}{96} = \frac{23+17\sqrt{17}}{96}$.

So, $A + 17B + 4C + 96D = 2 + 8\sqrt{17} + 8\ln(2) - 3 + 23 + 17\sqrt{17} = 22 + 8\ln(2) + 25\sqrt{17}$

#5 Mu Bowl
MAΘ National Convention 2019

Let

$$A = \int_1^2 \left(x^3 + \frac{1}{x^2}\right)\, dx \qquad B = \int_0^{\sqrt{\pi/3}} x\sin(x^2)\, dx$$

$$C = \int_0^{\pi/4} (\cos(x) + \sin(x))^2\, dx \qquad D = \int_1^2 \frac{1}{x^2+2x}\, dx$$

Find $\frac{A+C+e^{2D}}{B}$.

#5 Mu Bowl
MAΘ National Convention 2019

Solution:

$A = \int_1^2 \left(x^3 + \frac{1}{x^2}\right)\, dx = \left[\frac{1}{4}x^4 - \frac{1}{x}\right]_1^2 = \frac{17}{4}$.

$B = \int_0^{\sqrt{\pi/3}} x\sin(x^2)\, dx = \int_0^{\pi/3} \frac{1}{2}\sin(u)\, du = \left[-\frac{1}{2}\cos(u)\right]_0^{\pi/3} = -\frac{1}{4} + \frac{1}{2} = \frac{1}{4}$.

$C = \int_0^{\pi/4} (\cos(x) + \sin(x))^2\, dx = \int_0^{\pi/4} \cos^2(x) + \sin^2(x) + 2\sin(x)\cos(x)\, dx = \int_0^{\pi/4} 1 + \sin(2x)\, dx = \left[x - \frac{1}{2}\cos(2x)\right]_0^{\pi/4} = \frac{\pi}{4} + \frac{1}{2}$.

$D = \int_1^2 \frac{1}{x^2+2x}\, dx = \int_1^2 \frac{\frac{1}{2}}{x} - \frac{\frac{1}{2}}{x+2}\, dx = \frac{1}{2}\left[\ln\left(\frac{x}{x+2}\right)\right]_1^2 = \frac{1}{2}\ln\left(\frac{1}{2}\right) - \frac{1}{2}\ln\left(\frac{1}{3}\right) = \frac{1}{2}\ln\left(\frac{3}{2}\right)$.

So $\frac{A+C+e^{2D}}{B} = 4\left(\frac{17}{4} + \frac{\pi}{4} + \frac{1}{2} + \frac{3}{2}\right) = 17 + \pi - \sqrt{2} + 2 + 6 = 25 + \pi$.

#6 Mu Bowl
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Consider the polynomial

$$p(x) = rx^3 + (1-r)x^2 - r^2x + (r+1)^3$$

where r is a real number changing at a constant rate of +2 units per second.

In unit per second, let:

A be the rate of change of the sum of the roots when $r = 3$

B be the rate of change of the product of the roots when $r = 3$

Find $A - B$.

#6 Mu Bowl
MAΘ National Convention 2019

Solution:

The sum of the roots is $S = -\frac{(1-r)}{r} \to S' = \frac{1}{r^2}r' = \frac{2}{9}$.

The product of the roots is $P = -\frac{(r+1)^3}{r} = -r^2 - 3r - 3 - \frac{1}{r} \to P' = -2rr' - 3r' + \frac{r'}{r^2} = -12 - 6 + \frac{2}{9}$.

So $A - B = 18$.

#7 Mu Bowl
MAΘ National Convention 2019

Let

$$A = \int_1^{2019} \frac{x^{2019}}{x^{2019} + (2020 - x)^{2019}}\, dx \qquad\qquad B = \int_0^{\pi/2} \frac{1}{\cos(x) + \sin(x) + 2}\, dx$$

$$C = \int_0^1 \frac{x - 1}{\ln(x)}\, dx$$

It turns out $2A + B + C = pq + \sqrt{p}\arctan(\sqrt{p}) - \sqrt{p}\arctan\left(\frac{1}{\sqrt{p}}\right) + \ln(p)$ for a prime number p. Find q.

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Solution:

$A = \int_1^{2019} \frac{x^{2019}}{x^{2019}+(2020\quad)^{2019}}\, dx = \int_1^{2019} \frac{(2020-x)^{2019}}{x^{2019}+(2020-x)^{2019}}\, dx \to 2A = A + A =$
$\int_1^{2019} \frac{x^{2019}}{x^{2019}+(2020-\quad)^{2019}}\, dx + \int_1^{2019} \frac{(2020-x)^{2019}}{x^{2019}+(2020-x)^{2019}}\, dx = \int_1^{2019} \frac{x^{2019}+(2020-x)^{2019}}{x^{2019}+(2020-x)^{2019}}\, dx = \int_1^{2019} 1\, dx \to$
$2A = 2018 \to A = 1009$.

$B = \int_0^{\pi/2} \frac{1}{\cos(x)+\sin(x)+2}\, dx$. Let $t = \tan\left(\frac{x}{2}\right)$ so that $\cos(x) = \frac{1-t^2}{1+t^2}$, $\sin(x) = \frac{2t}{1+t^2}$, and $dx = \frac{2}{1+t^2}dt$. Then $\int_0^{\pi/2} \frac{1}{\cos(x)+\sin(x)+2}\, dx = \int_0^1 \frac{1}{\frac{1-t^2}{1+t^2}+\frac{2t}{1+t^2}+2}\frac{2}{1+t^2}dt = \int_0^1 \frac{2}{1-t^2+2t+2+2t^2}dt = \int_0^1 \frac{2}{t^2+2t+3}dt = \int_0^1 \frac{2}{t^2+2t+3}dt =$
$2\int_0^1 \frac{1}{(t+1)^2+2}dt = \sqrt{2}\arctan(\sqrt{2}) - \sqrt{2}\arctan\left(\frac{1}{\sqrt{2}}\right)$.
$C = \int_0^1 \frac{x-1}{\ln(x)}\, dx \to C(a) = \int_0^1 \frac{x^a-1}{\ln(x)}\, dx \to C'(a) = \int_0^1 \frac{\ln(x)x^a}{\ln(x)}\, dx = \int_0^1 x^a\, dx = \frac{1}{a+1} \to C(a) = \ln(a+1) + K$. When $a = 0$, $C(0) = \int_0^1 \frac{x^0-1}{\ln(x)}\, dx = 0 \to K = 0$. Therefore the desired integral is $C(1) = \ln(2)$.
Since $p = 2$ can be gotten from either B or C we don't actually need to do both. Then $q = A = 1009$.

#8 Mu Bowl
MAΘ National Convention 2019

Let R denote the finite region bounded by the x-axis and the curve $y = 3 - 3x^2$.

Let A be the area of R.

Let B be the volume of the solid formed when R is rotated about the x-axis.

Let C be the volume of the solid formed when R is rotated about the line $x = 1$.

Let D be the volume of the solid formed when R is rotated about the line $y = x - 1$.

Find $\frac{C}{A} + \frac{D}{B}$.

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Solution:

To do part D we're going to need to use the Theorem of Pappus, so we may as well use it for parts B and C as well. $A = \int_{-1}^{1} 3 - 3x^2\,dx = 2[3x - x^3]_0^1 = 2(3-1) = 4$. The x-coordinate of the centroid of region R is obviously zero by symmetry. The y-coordinate is $\frac{1}{2A}\int_{-1}^{1}(3-3x^2)^2\,dx = \frac{9}{A}\int_0^1(1 + x^4 - 2x^2)\,dx = \frac{9}{A}\left[x + \frac{1}{5}x^5 - \frac{2}{3}x^3\right]_0^1 = \frac{9+\frac{9}{5}-6}{A} = \frac{24}{5A} = \frac{6}{5}$. So for each part:

B: The distance from the centroid to the line is $\frac{6}{5}$. Therefore the volume is $2\pi\left(\frac{6}{5}\right)(4) = \frac{48}{5}$.

C: The distance from the centroid to the line is 1. Therefore the volume is $2\pi(1)(4) = 8\pi$.

D: The distance from the centroid to the line is $\frac{\left|0-\frac{6}{5}-1\right|}{\sqrt{2}} = \frac{11\sqrt{2}}{10}$. Therefore the volume is $2\pi\left(\frac{11\sqrt{2}}{10}\right)(4) = \frac{44\sqrt{2}\pi}{5}$.

$\frac{C}{A} + \frac{D}{B} = \frac{8\pi}{4} + \frac{44\sqrt{2}}{48} = 2\pi + \frac{11}{12}\sqrt{2}$.

Let L represent the line tangent to the curve $x^2y - xy^2 - x + y = 9$ at the point $(1,2)$.

Let M represent the line tangent to the curve $\begin{cases} x(t) = e^t - 2t \\ y(t) = t^2 - \ln(t+1) \end{cases}$ at $t = 0$.

If (A, B) is the point of intersection between L and M, find $A + B$.

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Solution:

$x^2y - xy^2 - x + y = 9 \to 2xy + x^2y' - y^2 - 2xyy' - 1 + y' = 0 \to y' = \frac{y^2-2xy+}{x^2-2xy+1} = \frac{4-4+1}{1-4+1} = -\frac{1}{2}$. So the line L is$y - 2 = -\frac{1}{2}(x-1) \to y = -\frac{1}{2}x + \frac{5}{2}$.

$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2t-\frac{1}{t+1}}{e^t-2} = -\frac{1}{1-2} = 1$. So the line M is $y = x - 1$.

The point of intersection is $-\frac{1}{2}x + \frac{5}{2} = x - 1 \to \frac{7}{2} = \frac{3}{2}x \to x = \frac{7}{3} \to y = \frac{4}{3} \to A + B = \frac{11}{3}$.

#10 Mu Bowl
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Let

$$A = \int_{-1}^{0} e^x \arctan(x+1)\ dx$$

$$B = \int_{\pi/4}^{\pi/3} \sec(x)\tan^2(x)\ dx$$

$$C = \int_{\pi/4}^{\pi/3} \sec^3(x)\ dx$$

$$D = \int_{-1}^{0} \frac{e^x}{x^2+2x+2}\ dx$$

Find $A + B + C + D$.

#10 Mu Bowl
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Solution:

$A + D = \int_{-1}^{0} e^x \arctan(x+1)\ dx + \int_{-1}^{0} \frac{e^x}{x^2+2x+2}\ dx = \int_{-1}^{0} (e^x)' \arctan(x+1) + e^x(\arctan(x+1))'\ dx = \int_{-1}^{0} [e^x \arctan(x+1)]'\ dx = e^0 \arctan(1) - e^{-1}\arctan(0) = \frac{\pi}{4}$.

$B + C = \int_{\pi/4}^{\pi/3} \sec(x)\tan^2(x)\ dx + \int_{\pi/4}^{\pi/3} \sec^3(x)\ dx = \int_{\pi/4}^{\pi/3} [\sec(x)\tan(x)]'\ dx = 2 * \sqrt{3} - \sqrt{2} * 1$.

So $A + B + C + D = \frac{\pi}{4} + 2\sqrt{3} - \sqrt{2}$.

Let $f(x) = 3x^4 - 2x^3 + x^2 - x + 2$

If

$$A = f(2) \qquad B = f'(2) \qquad C = \frac{f''(2)}{2!}$$

$$D = \frac{f'''(2)}{3!} \qquad E = \frac{f^{(4)}(2)}{4!} \qquad F = \frac{f^{(5)}(2)}{5!}$$

Find $A + B + C + D + E + F$.

#11 Mu Bowl
MAΘ National Convention 2019

Solution:

Since $f^{(n)}(2) = 0$ for $n > 5$ because $f(x)$ is a quartic, by Taylor's Theorem we have that

$$f(x) = f(2) + f'(2)(x-2) + \frac{f''(2)}{2!}(x-2)^2 + \frac{f'''(2)}{3!}(x-2)^3 + \frac{f^{(4)}(2)}{4!}(x-2)^4 + \frac{f^{(5)}(2)}{5!}(x-2)^5$$

Therefore the desired sum is just $f(3) = 3(81) - 2(27) + 9 - 3 + 2 = 197$.

#12 Mu Bowl
MAΘ National Convention 2019

Let $f(x) = 4x^3 - 2x + 2019$. Let R be the finite region bounded by $f(x)$, the x-axis, $x = 1$, and $x = 3$.

Let A be the value obtained when the area of R is approximated using a Left-handed Riemann Sum with 8 equal subintervals.

Let B be the value obtained when the area of R is approximated using a Right-handed Riemann Sum with 8 equal subintervals.

Let C be the value obtained when the area of R is approximated using the Trapezoidal Rule with 8 equal subintervals.

Let D be the value obtained when the area of R is approximated using Simpson's Rule with 8 equal subintervals.

Find $A + B - 2C + D$

#12 Mu Bowl
MAΘ National Convention 2019

Solution:

$\frac{A+B}{2} = C$ so $A + B - 2C = 0$.

Simpson's Rule is exact for cubics (and below) so $D = \int_1^3 (4x^3 - 2x + 2019)dx = [x^4 - x^2 + 2019x]_1^3 = 81 - 9 + 6057 - 1 + 1 - 2019 = 72 + 4038 = 4110$. This is also the final sum.

#13 Mu Bowl
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Let $A = 2019$ if the statement

"There exists a value of $c \in (1,3)$ such that the slope of $f(x) = \frac{1}{x-2}$ is -1"

is true, or -2019 if it is false.

Let $B = 2020$ if the statement

"A function may only cross its oblique asymptote a finite number of times"

is true, or -2020 if it is false.

Let $C = 2021$ if the statement

"There exists a function that is continuous and differentiable everywhere, but has a second derivative nowhere"

is true, or -2021 if it is false.

Find $A + B + C$

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Solution:

For A, the Mean Value Theorem does not apply because $f(x)$ is not continuous. We can check directly: $f'(x) = \frac{-1}{(x-2)^2} = -1 \rightarrow (x-2)^2 = 1 \rightarrow x = 3$ or $x = 1$. Neither of these are on the open interval so $A = -2019$.

For B, one of the many counter-examples is $y = x + \frac{\sin(x)}{x}$, so $B = -2020$.

For C, let $f''(x)$ be the Dirichlet function $D(x) = \begin{cases} 1 & x \in \mathbb{Q} \\ 0 & Else \end{cases}$. Then $f(x) = \int_{-\infty}^{x} \int_{-\infty}^{u} D(t)\, dt du$ will work. So $C = 2021$

$A + B + C = -2019 - 2020 + 2021 = -2018$.

Let

A be the area contained within the curve $\frac{(x-2)^2}{4}+\frac{(y+1)^2}{9}=1$

B be the area contained within the curve $|x|+|y|=20$

C be the area contained within the polar curve $r^2=3\sin(2\theta)$

D be the area contained within the polar curve $r=1-\cos(\theta)$

Find $A+B+C+D$.

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Solution:

A: The area of this ellipse is $2*3*\pi=6\pi$.

B: The area of this square with side length $20\sqrt{2}$ is 800.

C: The area of this lemniscate is $\frac{2}{2}\int_{-\pi/4}^{\pi/4} r^2 d\theta=3\int_{-\pi/4}^{\pi/4}\sin(2\theta)d\theta=3$.

D: The area of this cardioid is $\frac{1}{2}\int_0^{2\pi} r^2 d\theta=\frac{1}{2}\int_0^{2\pi}(1-\cos(\theta))^2 d\theta=\frac{3\pi}{2}$.

$A+B+C+D=803+\frac{15\pi}{2}$.

Nationals 2019 Mu Bowl ANSWERS

0) $\frac{e^{15}}{3}$

1) $647 + 3\ln(2)$

2) $\frac{8}{3}$

3) $5\ln(2) - \frac{1}{3}$

4) $22 + 8\ln(2) + 25\sqrt{17}$

5) $25 + \pi$

6) 18

7) 1009

8) $2\pi + \frac{11}{12}\sqrt{2}$

9) $\frac{11}{3}$

10) $\frac{\pi}{4} + 2\sqrt{3} - \sqrt{2}$

11) 197

12) 4110

13) -2018

14) $803 + \frac{15}{2}$