

Team Relay

2019 MAΘ National Convention

Question #0

Seat 1 – Theta

Solve for x : $2x + 5 = 11$

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Question #0

Seat 2 – Alpha

Evaluate $\cos \frac{\pi}{A}$.

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Question #0

Seat 3 – Mu

Given that $f(x) = 4x^2 + 3x + 2$, find the value of $f'(B)$.

Question #1

Seat 1 – Theta

Evaluate $(\log_2 16)(\log_{49} 7) + (\log_5 1331)(\log_{11} 625)$.

Question #1

Seat 2 – Alpha

Find the positive difference between the zeros of $f(x) = x^2 - Ax - 72$.

Question #1

Seat 3 – Mu

A particle moves along the x -axis so that its position at any time $t \geq 0$ is $x(t) = \frac{1}{12}t^3 + \frac{1}{4}t^2 + \ln t$, where t is measured in seconds.

What is the acceleration, in units per second squared, of the particle at $t = B$?

Question #2

Seat 1 – Theta

The focus of the parabola with equation $y^2 - 20x - 4y + 304 = 0$ has coordinates (p, q) .
What is the value of $(2p + 5q)$?

Question #2

Seat 2 – Alpha

An ellipse with center at $(1, A)$ has a horizontal major axis of length A , and an area of 100π .
The endpoints of the minor axis of the ellipse are $(1, r)$ and $(1, s)$, where $r < s$.

Find the value of s .

Question #2

Seat 3 – Mu

The area bounded by the graphs of $y = x + k$, $y = 0$, $x = 0$, and $x = k$ is the base of a solid.
Cross-sections of the solid perpendicular to the x -axis are squares. The volume of the solid is B .

Given that $k > 0$, find the value of k^3 .

Question #3

Seat 1 – Theta

Find the base-10 number which is equivalent to the base-6 number 12.3_6 .

Question #3

Seat 2 – Alpha

The row-2, column-2 entry of matrix $X^{-1}X^T$ is A . If $X = \begin{bmatrix} 1 & 2 \\ 3 & k \end{bmatrix}$, then what is the value of k ?

Question #3

Seat 3 – Mu

It is given that $\frac{dy}{dx} = x^2 - x^4$ and that $y = B$ when $x = 2$. What is the value of y when $x = 1$?

Question #4

Seat 1 – Mu

The values of twice-differentiable functions f , g , and their first and second derivatives are shown in the table for selected values of x .

x	f	f'	f''	g	g'	g''
2	-4	8	-1	3	10	-5
3	7	-20	-9	2	6	4

Evaluate $\frac{d}{dx}(f(g(x))) + \frac{d^2}{dx^2}(x^2 g(x))$ where $x = 3$.

Question #4

Seat 2 – Theta

Find the total number of positive integer factors of $\lfloor A \rfloor$, where A represents the greatest integer value of A .

Question #4

Seat 3 – Alpha

Find the sum of the digits of the base-10 representation of $\left| \left| (1 + i\sqrt{3})^B \right| \right|$, where $i = \sqrt{-1}$ and where $\lfloor k \rfloor$ represents the greatest integer function of k .

Question #5

Seat 1 – Mu

Given $f(x) = x^2 + \sqrt[3]{x}$ with domain $x \geq 0$, let L be the line tangent to the graph of f at the point where $x = 1$. Using L , find the y -value that represents an approximation to the function f where its exact y -value is 66.

Question #5

Seat 2 – Theta

A regular $[A]$ -gon has exterior angles (one at each vertex) whose degree measures form an arithmetic sequence with a common difference of 1. The smallest exterior angle has a measure of k° .

Find the value of k .

Question #5

Seat 3 – Alpha

Determine the number of x -intercepts of the graph of $y = \sin Bx$ on the interval $0 \leq x \leq 2\pi$.

Question #6

Seat 1 – Mu

The region bounded by the graphs of $y = \frac{1}{\sqrt{1-x^2}}$, $y = 2 - x$, $x = 0$, and $x = \frac{1}{2}$ has an area of $P + Q\pi$ for rational numbers P and Q . Find the value of $P + Q$.

Question #6

Seat 2 – Theta

The value of x such that $\frac{x}{1 + \frac{x}{1 + \frac{x}{1 + \dots}}} = |A|$ can be written as a positive fraction $\frac{P}{Q}$, where P and Q are relatively prime positive integers. Find the value of $P + Q$.

Question #6

Seat 3 – Alpha

Let k be the sum of the digits of the base-10 representation of B .

Find the value of $\sin\left(\frac{k\pi}{3}\right) + \sin\left(\frac{B\pi}{3}\right)$.

Question #7

Seat 1 – Alpha

Evaluate $\lim_{x \rightarrow \infty} \frac{3x+2+2\sin x}{x}$.

Question #7

Seat 2 – Mu

The right-sided Riemann sum approximation of $\int_1^A (x \ln x) dx$ using two equal subdivisions is equal to $\ln k$ for some positive integer k . What is the value of k ?

Question #7

Seat 3 – Theta

Let s equal the sum of the digits of the base-10 representation of B .

An infinite geometric sequence has first term s and sum B . Find the common ratio of the sequence.

Question #8

Seat 1 – Alpha

Given vectors $\mathbf{v} = 2\mathbf{i} - 5\mathbf{j} + 4\mathbf{k}$ and $\mathbf{w} = 7\mathbf{i} - 6\mathbf{j} - 9\mathbf{k}$:

If $\mathbf{v} \times \mathbf{w} = p\mathbf{i} + q\mathbf{j} + r\mathbf{k}$ for scalars p, q, r , then find the value of $(\mathbf{v} \cdot \mathbf{w}) + p - q - r$.

Question #8

Seat 2 – Mu

Given the function $f(x) = A\sqrt{x}$ with domain $[0, A]$:

Find the value of c in the interval $(0, A)$ that satisfies the conclusion of the Mean Value Theorem for the function f on the interval $[0, A]$.

Question #8

Seat 3 – Theta

Find the sum of the coefficients of the first $(B^2 - 1)$ terms of the expansion of $(Bx - 1)^6$.

Question #9

Seat 1 – Alpha

Find the number of positive integral divisors of the constant term of the expansion of

$$\left(x^2 + \frac{2}{x}\right)^6$$

Question #9

Seat 2 – Mu

A particle moves along the x –axis so that at any time $t \geq 0$, with t measured in seconds, its velocity v in units per second is given by $v(t) = \left(t - \frac{A}{10}\right)\left(t - \frac{A}{5}\right)$. Find the total distance traveled by the particle from $t = 0$ to $t = \frac{A}{4}$.

Question #9

Seat 3 – Theta

The slope of a line through the points $(1, B)$ and (B, k) is equal to B . Find the value of k .