SPEED MATH

2019 Mu Alpha Theta National Convention

1.	Find the sum of the last three digits of 995 ³ .
2.	What is the maximum area of a right triangle with hypotenuse of length 8?
3.	We call a polygon "roundish" if more than half of its angles are greater than 170°. A "roundish" polygon must have at least how many sides?
4.	Find $f(1) + f(-1)$ if f is given by the following: $f(x) = x^2 \sin x + 2019$.
5.	How many positive integers less than 1000000 have digital sum of 6?
6.	The number of zeros at the end of 2019! added to the sum of the positive integral factors of 2019 is?
7.	What is the maximum number of games needed to be played in a double- elimination tournament with 2019 players to determine a single winner?
8.	Every 10 minute interval, the Doctor chooses to drive at 60mph with probability $\frac{3}{5}$ for the interval, or else he decides to sleep for the entire interval. If the Car-o-Line Service drives at 45mph consistently, what is the probability that the Doctor finishes a 30 mile drive no later than Car-o-Line?
9.	What is the minimum value of $ 1 - 3x + 2x - 7 $?
10.	How many 3-digit positive integers are divisible by 3 and contain the digit 2?
11.	A regular hexagon has area $6\sqrt{3}$. Find the area of the circle that circumscribes it.
12.	Amy and Anthony are playing a game in which they are taking coins from a stack of 2019 coins. They can take any integer number of coins from 1 to 10 on their turn, and the winner is decided by who takes the last coin. If Amy goes first, how many should she take to guarantee victory under optimal play?

Solve for *x*: |2 - |x - 3|| = 1

- ____14. A sequence *S* is defined with first term 0 and each following term given by $S_{a+1} = \frac{S_a}{2019} + 2018$. Find $\lim_{x \to \infty} S_x$.
 - _15. If a + b = 5, b + c = 1, and c + a = 6, then find the value of a + b + c.
- ___16. Find the sum of the first 10 perfect squares.
- _17. A cube is inscribed in a sphere. Find the ratio of the cube's volume to the sphere's volume.

Evaluate:
$$\sum_{k=4}^{\infty} \frac{-1}{k^2 - 5k + 6}$$

_18.

20.

25.

- 19. A positive integer *x* has remainder 3 when divided by 5, remainder 4 when divided by 7, and remainder 6 when divided by 11. What is the smallest possible value for *x*?
 - Two fair six-sided dice are rolled. What is the probability that the sum of the rolls is greater than 7?
- 21. A positive integer is "neighborly" if none of its digits differ from any other digit by more than 2. A positive integer is "sloped" if the digits from left-to-right are all in ascending or descending order. (Sloped numbers can have digit ties.) How many three-digit numbers are neighborly but not sloped?
- 22. Evaluate the expression when x = 2: $x^{2019} - 4x^{2018} + 5x^{2017} + 3x - 2^{2017}$
- 23. David likes drawing concentric circles. He starts with a circle of radius 2. For each following concentric circle, he makes sure that the area between that circle and the previous one is 4π . What is the radius of the 2019th circle drawn?
- 24. Simplify the following without *i* in the denominator: $\frac{1-i}{(5-i)(1+i)}$
 - A $2\pi \times 4\pi$ sheet of paper is presented to Michael, who promptly rolls it to make a cylinder. Find the positive difference between the volumes of the two possible cylinders created (when rolled on each of the sides with no overlap).

__13.