For all questions, answer choice "E. NOTA" means none of the above answers is correct.
Let $[R]$ denote the area of region $R$. For example, $[ABC]$ denotes the area of triangle $ABC$.
Additionally, figures are not drawn to scale.
Good luck, and have fun! :)

**(1)** What is the area of an equilateral triangle with side length 4?

A. $4\sqrt{3}$ B. 8 C. 16 D. $16\sqrt{3}$ E. NOTA

**(2)** What is the area of a square with diagonal 4?

A. 32 B. 16 C. 8 D. $16/3$ E. NOTA

**(3)** What is the area of a regular octagon with side length 4?

A. 16 B. $16(1+\sqrt{2})$ C. 32 D. $32(1+\sqrt{2})$ E. NOTA

**(4)** What is the area of a rhombus with side length 4?

A. 16 B. $8\sqrt{3}$ C. $8\sqrt{2}$ D. 8 E. NOTA

**(5)** What is the area of an ellipse with major axis of length 5 and minor axis of length 3?

A. $15\pi$ B. $\frac{15\pi}{2}$ C. $\frac{15\pi}{4}$ D. $\frac{15\pi}{8}$ E. NOTA

**(6)** What is the volume of a cube with space diagonal 4?

A. 8 B. 64 C. $\frac{8\sqrt{3}}{9}$ D. $\frac{64\sqrt{3}}{9}$ E. NOTA

**(7)** What is the volume of a regular octahedron with space diagonal 4?

A. $\frac{32}{3}$ B. $\frac{64\sqrt{3}}{9}$ C. $\frac{64\sqrt{2}}{3}$ D. 64 E. NOTA

**(8)** What is the volume of the ellipsoid formed by rotating an ellipse with major axis of length 6 and minor axis of length 4 about its minor axis?

A. $16\pi$ B. $18\pi$ C. $64\pi$ D. $144\pi$ E. NOTA

**(9)** Brooke is doing laundry. The clothes compartment of each washing machine is a cylinder with radius 20 inches and height 40 inches. For a dryer, its compartment is a cylinder with radius 30 inches and height 40 inches.

Unfortunately, the dryers are poorly designed: they will only dry clothing if the equality $pt = 60$ is satisfied, where $p$ is the proportion of the dryer compartment's volume that is filled with clothes, and $t$ is the time in minutes that the dryer runs.

Brooke fills two washing machines 75% full. Assuming Brooke's clothing does not change volume after being washed, and that she puts all her clothing into a single dryer, for how long (in minutes) does she have to run the dryer for her clothing to be dried?

A. 60 B. $\frac{135}{2}$ C. 90 D. 135 E. NOTA

**(10)** What is the largest area that can be enclosed by a wire of length 2?

A. 1 B. $1/\pi$ C. $1/\pi^2$ D. $1/\pi^3$ E. NOTA

**(11)** Romi wants to make a rectangular pigpen in the Math Lounge. Romi has 160 $ft$ of fencing, and he is using one wall as one of the boundaries of the pigpen. What is the largest area that Romi's pigpen can have, in $ft^2$? (Assume the lounge is sufficiently large.)

A. 1600 B. 3200 C. 6400 D. 12800 E. NOTA

**(12)** A regular hexagon is inscribed in a circle of radius 6. What is the area of the hexagon?

A. $9\sqrt{3}$ B. $36\sqrt{3}$ C. $54\sqrt{3}$ D. $72\sqrt{3}$ E. NOTA

**(13)** What is the area of the quadrilateral with vertices $(1,2)$, $(0,5)$, $(6,1)$, and $(-1,3)$ on the coordinate plane?

A. 11/4 B. 11/2 C. 11 D. 22 E. NOTA

**(14)** What is the resulting volume when an isosceles triangle with side lengths 5, 5, and $\sqrt{10}$ is revolved 360° about one of its longer sides?

A. $\frac{25\pi\sqrt{10}}{3}$ B. $15\pi$ C. $\frac{50\pi}{3}$ D. $25\pi$ E. NOTA

**(15)** Let $V_0$ be the volume of a given sphere. Let $V_1$ be the volume of the sphere after it is scaled so that the area of a great circle of the sphere is quadrupled. Compute $\frac{V_1}{V_0}$.

A. 8 B. 16 C. 32 D. 64 E. NOTA

**(16)** Two circles of different radii are concentric. Given that the length of a chord of the larger circle that is tangent to the smaller circle is 8, compute the area of the annulus between the circles.

A. $9\pi$ B. $12\pi$ C. $16\pi$ D. $25\pi$ E. NOTA

**(17)** A circular dartboard is composed of 3 concentric circles. The largest one with radius 3 forms the boundary of the dartboard, and the smaller two with radii 1 and 2 delineate three regions: an outer annulus, an inner annulus, and a circle in the middle.

Alec is playing darts by the following rules:

- If his dart hits the region between the boundary of the dartboard and the circle with radius 2, Alec wins 1 point.
- If his dart hits the region between the circles of radii 1 and 2, Alec wins 4 points.
- If his dart hits the middle circle, he wins 10 points.

Given that Alec hits the dartboard, and his dart hits any point of the dartboard with equal probability, what is the expected number of points Alec receives with one throw?

A. 1 B. 2 C. 3 D. 4 E. NOTA

**(18)** Erin the ant is walking in a circle on the ground. She is trying to find a good angle to see what her economics final grade is. Erin's economics final grade is posted 15 units directly above the center of Erin's walking path. Disregarding the mass of the final grade, if Erin stays a constant 17 units away from her grade (she's neither good at economics nor at finding a better spot), what is the area enclosed by her walking path?

A. $25\pi$ B. $40\pi$ C. $51\pi$ D. $64\pi$ E. NOTA

**(19)** Point $C$ lies on a circle. Let point $C'$ be the reflection of point $C$ across the diameter $AT$, where $m\angle CAC' = 60°$. Given that $AT = 4$, what is the area bounded by chords $AC$ and $AC'$, and arc $CTC'$?

A. $\frac{8\pi}{3}$ B. $2\sqrt{3} + \frac{4\pi}{3}$ C. $\frac{4\pi}{3} - \sqrt{3}$ D. $4\sqrt{3}$ E. NOTA

**(20)** What is the maximum number of regions that 4 lines can split a plane into?

A. 7 B. 8 C. 11 D. 12 E. NOTA

**(21)** A plane cuts a unit cube into two pieces. Three vertices of the cube lie on the plane, and no edges of the cube lie on the plane. What is the volume of the piece with the larger volume?

A. $\frac{1}{2}$ B. $\frac{2}{3}$ C. $\frac{3}{4}$ D. $\frac{5}{6}$ E. NOTA

**(22)** What is the volume of a right hexagonal frustum with a height of 1 and regular hexagonal bases of areas $54\sqrt{3}$ and $96\sqrt{3}$?

A. $54\sqrt{3}$ B. $74\sqrt{3}$ C. $75\sqrt{3}$ D. $128\sqrt{3}$ E. NOTA

**(23)** In a cyclic trapezoid, the bases measure 5 and 10. One of the legs measures 5. What is the area of the trapezoid's circumscribed circle?

A. $25\pi$ B. $\frac{225\pi}{4}$ C. $100\pi$ D. Not enough information E. NOTA

**(24)** In triangle $MEO$, $ME = 6$, $EO = 7$, and $OM = 8$. Let $X_1, \ldots, X_{99}$ be evenly spaced on side $OM$. Compute the ratio $\frac{[X_{21}EX_{84}]}{[X_2EX_5]}$.

A. $\frac{63}{4}$ B. 16 C. 21 D. $\frac{64}{3}$ E. NOTA

**(25)** Ellen and Kim want to meet at the Hotelling's Lotel to study economics. They arrive independently at uniformly distributed times between 9 pm and 11 pm on that day. Ellen and Kim both agree to wait 30 minutes for the other to show up.

Let $P$ be the probability that Ellen and Kim get to study together. Let $E$ be the probability that Ellen arrives first. Compute $\frac{P}{E}$.

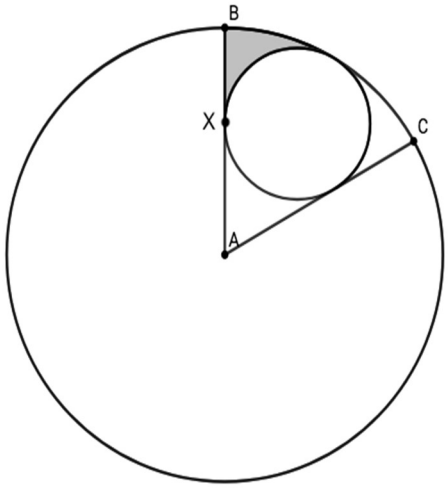
A. $\frac{1}{8}$ B. $\frac{7}{8}$ C. 1 D. $\frac{9}{8}$ E. NOTA

**(26)** Five points can be placed on a plane such that the distances between the 10 pairs of points are $1,1,1,1,\sqrt{2},\sqrt{2},2,2,\sqrt{5},\sqrt{5}$. Let these five points form the set $S$. Let $S^*$ be the set of all intersection points between segments formed by pairs of points in $S$.

There are $n$ triangular regions with positive area and vertices that are distinct points in the union $S \cup S^*$. What is the value of $n$?

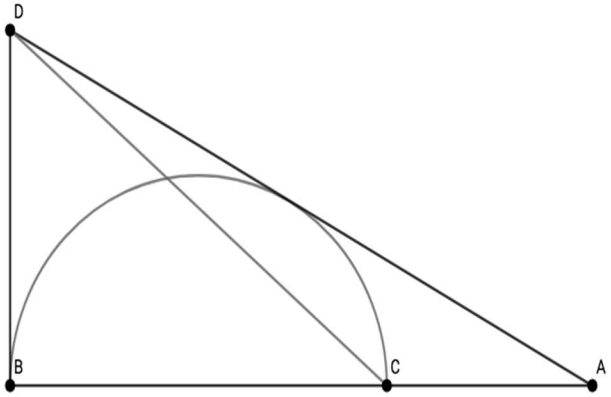
A. 4 B. 7 C. 10 D. 27 E. NOTA

**(27)** In the figure, $m\angle BAC = 60°$. There is a circle inscribed in the $60°$ sector tangent to $\overline{AB}$ at point $X$. Given that $AX = 3$, find the area of the shaded region, which is bound by the larger circle, $\overline{AB}$, and the smaller circle.



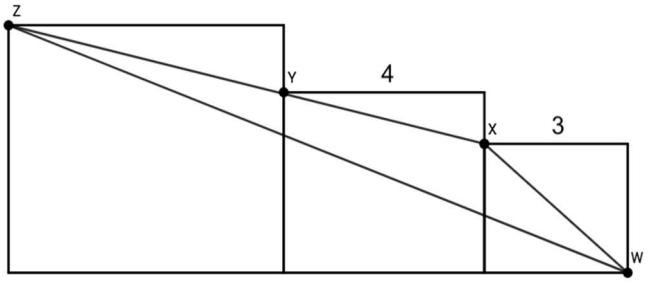

A. $\pi - \frac{3\sqrt{3}}{4}$
B. $\frac{5\pi}{4} - \frac{3\sqrt{3}}{2}$
C. $\frac{9\pi - 9\sqrt{3}}{4}$
D. $4\pi - 3\sqrt{3}$
E. NOTA

**(28)** In right triangle $ABD$ with right angle at vertex $B$, construct a semicircle tangent to $\overline{AD}$, which intersects $\overline{AB}$ at $B$ and $C$. Given that $BD = BC = 1$, compute $[ACD]$.




A. $\frac{1}{2}$ B. $\frac{1}{3}$ C. $\frac{1}{4}$ D. $\frac{1}{5}$ E. NOTA

**(29)** Three squares are adjacent. The small square has side length 3 and the medium square has side length 4. Let $X$, $Y$, $Z$ be the upper left vertex of the small, medium, and large square respectively, and let $W$ be the bottom right vertex of the smallest square. Given $Y$ lies on $XZ$, compute $[WXZ]$.

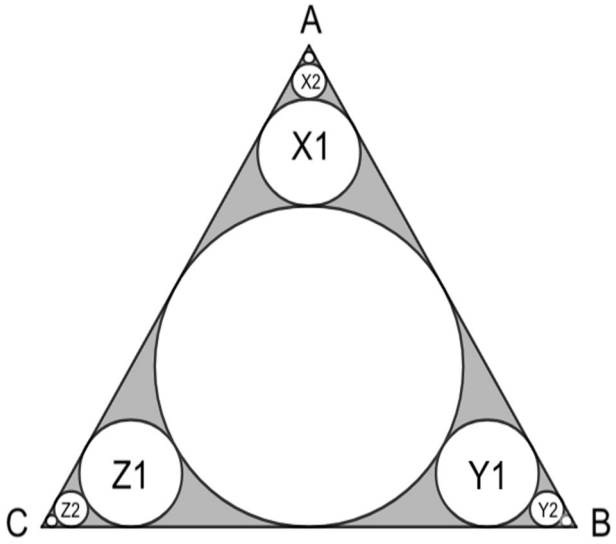



A. $\frac{65}{18}$
B. $\frac{21}{2}$
C. $\frac{29}{2}$
D. Not enough information
E. NOTA

**(30)** In an equilateral triangle, an infinite number of circles are constructed by the following process, which is repeated infinitely.

0 A circle is inscribed.
1 Construct circles $X1, Y1, Z1$ as follows:
   a. $X1$ is tangent to the inscribed circle and sides $AB$, $AC$.
   b. $Y1$ is tangent to the inscribed circle and sides $BA$, $BC$.
   c. $Z1$ is tangent to the inscribed circle and sides $CA$, $CB$.
2 Construct circles $X2, Y2, Z2$ as follows:
   a. $X2$ is tangent to $X1$ and sides $AB$, $AC$.
   b. $Y2$ is tangent to $Y1$ and sides $BA$, $BC$.
   c. $Z2$ is tangent to $Z1$ and sides $CA$, $CB$.
n Construct circles $Xn, Yn, Zn$ as follows:
   a. $Xn$ is tangent to $X(n-1)$ and sides $AB$, $AC$.
   b. $Yn$ is tangent to $Y(n-1)$ and sides $BA$, $BC$.
   c. $Zn$ is tangent to $Z(n-1)$ and sides $CA$, $CB$.

Given that $BC = 2$, compute the area of the triangle not in any of the circles, shaded in the figure below.




A. $\sqrt{3} - \frac{11\pi}{24}$ B. $\sqrt{3} - \frac{\pi}{2}$ C. $\sqrt{3} - \frac{2\pi}{3}$ D. $\sqrt{3} - \frac{5\pi}{6}$ E. NOTA