

Theta ciphering nationals 2019 solutions

$$\#0 - \frac{-4}{n} = \frac{3}{n-1} \rightarrow -4n + 4 = 3n \rightarrow 7n = 4 \rightarrow n = \frac{4}{7} \quad 7n=4$$

$$1. \quad (x^2 + 2x)^2 = 45^2 \rightarrow x^4 + 4x^3 + 4x^2 = 2025$$

$$2025 + 45 - 70 = 2000$$

$$2.. \quad 13!(15 \cdot 14 - 1) = 13!(209) = 13! \cdot 11 \cdot 19$$

$$11 + 13 + 19 = 43$$

3. Call the roots  $r-d$ ,  $r$ , and  $r+d$ . sum of roots is  $3r=12$ . Therefore  $r=4$  is a root. Plug in and you get  $64-192+168+K=0$   $K=-40$

4.  $(1+2x-x^2)(1+2x-x^2)(1+2x-x^2)(1+2x-x^2)$ . The only way to get  $x^7 \rightarrow (-x^2)(-x^2)(-x^2)(2x)$ . This will occur 4 times so  $(-2)(4) = -8$

$$\frac{3}{U} - \frac{4}{M} = -8 \quad \frac{-6}{U} + \frac{8}{M} = 16$$

$$5. \quad \frac{2}{U} + \frac{7}{M} = 43 \quad \frac{6}{U} + \frac{21}{M} = 129$$

$$\frac{29}{M} = 145 \rightarrow M = \frac{1}{5} \rightarrow U = \frac{1}{4} \rightarrow 20$$

$$6. \quad \frac{2019}{5} < \frac{L+U}{L} < \frac{2019}{4} \rightarrow \frac{2014}{5} < \frac{U}{L} < \frac{2015}{4}$$

$$402 \frac{4}{5} < \frac{U}{L} < 503 \frac{3}{4} \rightarrow 101$$

$$1000 < 5x+1 < 2019 \rightarrow 999 < 5x < 2018$$

$$7. \quad 199 \frac{4}{5} < x < 403 \frac{3}{5} \rightarrow 403 - 200 + 1 = 204$$

8.  $U = 199 \rightarrow L < 796 \rightarrow L = 795 \rightarrow R < 2385$   
 $R = 2384 \rightarrow M < 4768 \rightarrow M = 4767$

9. Draw a picture. Call the incenter point J. Triangles UFJ and LJW are both isosceles triangles. Therefore segment UF is congruent to JF and segment JW is congruent to segment WL. The area of the triangle is therefore  $27+15=42$

10. Call the side of the first square 2. The radius of the circle is therefore  $\sqrt{5}$ . The diameter is  $2\sqrt{5}$

$2\sqrt{5}$ . The side of the bigger circle is  $\frac{2\sqrt{5}}{\sqrt{2}} \rightarrow area = \left(\frac{2\sqrt{5}}{\sqrt{2}}\right)^2 = 10$

$$\frac{4}{10} = \frac{2}{5} \rightarrow 2+5 = 7$$

11. There are 3 solutions: (7,4), (1,4), and (-1,-4). The x-coordinates sum to:  $7+1-1=7$

12.  $\frac{2015}{5} \rightarrow 403 \rightarrow \frac{403}{5} \rightarrow 80 \rightarrow \frac{80}{5} \rightarrow 16 \rightarrow \frac{16}{5} \rightarrow 3$ . You do not pick up another zero until you

$403+80+16+3 = 502$   
 get to 2020. Therefore 2019 is the largest number. 2019

Answers:

0. 4

1. 2000

2. 43

3. -40

4. -8

5. 20

6. 101

7. 204

8. 4767

9. 42

10. 7

11. 7

12. 2019