"For all questions, answer choice "E. NOTA" means none of the above answers is correct."

1. The area of a circle in square yards is the same as the circumference of the circle in feet. What is the radius in feet?

A. 2 B. 6 C. 12 D. 18 E. NOTA

Answer D. Let the radius in yards be r. Then $3 \cdot 2\pi r = \pi r^2$, so $r = 6$, or 18 feet.

For #2-3 us the following two circles:

$$x^2 + (y-4)^2 = 16$$
$$x^2 + (y-4)^2 = 32$$

2. Which of the following gives an accurate relationship between the two circles:

A. The radius of the second is 16 units larger than that of the first
B. The radius of the second is 4 units larger than that of the first
C. The center of the second is 16 units higher than that of the first
D. The radius of the second is twice as large as that of the first
E. NOTA

Answer E. The center is $(0,4)$ for both circles, but the radius of the second is $4\sqrt{2}$ whereas the radius of the first is 4.

3. What is the area between the two circles?

A. $\left(4\sqrt{2}-4\right)^2\pi$ B. 4π C. 16π D. 256π E. NOTA

Answer C. The area of the second is 32π and the area of the first is 16π.

For #4-7 use the following function: $f(x) = \sqrt{-x^2 + 18x + 19}$

4. If I formed an isosceles triangle with one side on the x-axis and all its vertices on the function, what would be the sum of the base angles?

A. 30 B. 60 C. 90 D. 120 E. NOTA

Answer C. Since the function is a semicircle, the inscribed angle must be 90, hence the sum of the other two angles will also be 90.

5. What is the area of the triangle?

A. $\frac{81}{2}$ B. 81 C. 100 D. 162 E. NOTA

Answer C. The function may be re-written $f(x) = \sqrt{100 - (x-9)^2}$ which is a semi-circle of radius 10, which represents both the height of the triangle and half the base, yielding an area of 100.

6. If I were to create a scalene triangle with the same conditions, what is the probability that the new triangle's area would be greater than that of the isosceles triangle?

A. 0 B. $\frac{4}{9}$ C. $\frac{5}{9}$ D. 1 E. NOTA

Answer A. All triangles formed in this way would have the same base, so it is only the height that would differ. Since the isosceles triangle has the point opposite the x-axis highest on the semi-circle, no other triangle under the same conditions could have a larger area. Therefore the probability is 0.

7. What is the surface area of the solid generated by taking the region enclosed by this function and the x-axis and rotating it about the line $x = 9$?

A. 200π B. 250π C. 300π D. 400π E. NOTA

Answer C. The solid would be a hemisphere of radius 10. The circular surface on the bottom is a circle of radius 10. That surface has an area of 100π and the hemispherical surface has an area of $2\pi r^2 = 200\pi$.

8. If a circle of radius r is drawn on the complex plane with its center at the complex number $2r + ri$, how many points on the circle are real numbers?

A. 0 B. 1 C. 2 D. depends on the value of r E. NOTA

Answer B. The center of the circle would be r units above the real axis, which would lie tangent to the bottom of the circle.

9. If the points $A(-2,1),\ B(-2,5),\ and\ D(3,-1)$ are on circle C, what is the radius of circle C?

A. $\frac{17}{10}$ B. $\frac{\sqrt{29}}{2}$ C. 3 D. $\frac{\sqrt{1769}}{10}$ E. NOTA

Answer D. We can find C using the perpendicular bisectors of the chords AB and BD. The first bisector is easy: $y = 3$. The second is a little more tedious. The slope of BD is $-\frac{6}{5}$ so the slope of the bisector will be $\frac{5}{6}$. The midpoint is $(\frac{1}{2}, 2)$ therefore the equation of the second bisector is: $y = \frac{5}{6}x + \frac{19}{12}$. This makes the center $C(\frac{17}{10}, 3)$ and the radius $\frac{\sqrt{1769}}{10}$.

10. Assuming it is not degenerate, under what conditions must the following conic be a circle: $Ax^2 + Cy^2 + Dx + Ey + F = 0$

A. $AC > 0$ B. $DE > 0$ C. $F < 0$

D. $A = C$ E. NOTA

Answer D.

11. Give the radius of the following circle: $9x^2 + 9y^2 + 36x + 36y - 189 = 0$

A. $\sqrt{13}$ B. $\sqrt{21}$ C. 5 D. $\sqrt{29}$ E. NOTA

Answer D. The equation may be rewritten, after completing the square in x and y, as $(x+2)^2 + (y+2)^2 = 29$, giving us $\sqrt{29}$ as the radius.

12. Find $|ab + cd|$ if (a,b) and (c,d) represent the intersection points between the circle in #11 above and the following circle: $9x^2 + 9y^2 + 36x + 81y - 189 = 0$

A. 0 B. 5 C. 9 D. 45 E. NOTA

Answer A. Subtracting the two equations yields $y = 0$, at which point the value of the expression is obvious.

13. Two angles $0 < \alpha < \beta < 180°$ are such that $sin\alpha = sin\beta$, which of the following must be true?

A. $\alpha + \beta = 90$ B. $\alpha + \beta = 180$ C. $\beta - \alpha = 90$ D. $cos\alpha = cos\beta$

E. NOTA

Answer B. This implies that the y-coordinates of the points of intersection with the unit circle are the same, giving us an equal angle of elevation off of the positive and negative x-axis. Both of those angles of elevation are α, so therefore the angle β, measure from the positive x-axis, must be a supplement of α.

14. Let $(x_1, y_1), (x_2, y_2), \dots, (x_n, y_n)$ be points on the unit circle. Find $\sum_{i=1}^{n} x_i^2 + y_i^2$ in terms of n

A. 1 B. n C. $2n$ D. n^2 E. NOTA

Answer B. The summand is the square of the distance from the point to the origin, which is 1 for all the points on the unit circle.

15. If Sphere A has 64 times the volume of Sphere B, then how many times larger is the radius of sphere A than the radius of sphere B?

A. 4 B. 8 C. 16 D. 64 E. NOTA

Answer A. This implies that $r_A^3 = 64r_B^3$. Cube-rooting both sides yields answer A.

16. A circle is inscribed in ΔABC and three chords are drawn between the three points of tangency, forming three smaller triangles that share a vertex with ΔABC. What kind of triangles must all three of these be?

A. Equilateral B. Isosceles C. Obtuse D. Right E. NOTA

Answer B. Each of these triangles would be composed of two tangent segments, which must be congruent.

For problems 17-19 use the following: Circle C lies on the coordinate plane with points A(-4, -14) and B(2,-14) on the circle. The equation of a tangent to circle C is y = ¾ x – 11.

17. Find the coordinates of C.

A. $(-1,-18)$ B. $(-1,-14)$ C. $(-1,-11)$ D. $(-1,-10)$ E. NOTA

Answer A. Plugging in the coordinates for A into the tangent line equation shows us that the line is tangent to the circle at point A. The center must then lie on the line perpendicular to the tangent line at this point. The equation for the perpendicular line is $y = -\frac{4}{3}x - \frac{58}{3}$. The center must also lie on the perpendicular bisector of the chord AB, the equation for which is $x = -1$. The center must be the intersection of these, which is $(-1,-18)$.

18. Let D be another point on the circle such that AD is a diameter, write the equation of the tangent at point D.

A. $y = -\frac{4}{3}x - \frac{58}{3}$ B. $y = -\frac{4}{3}x - \frac{50}{3}$ C. $y = \frac{3}{4}x - \frac{39}{2}$ D. $y = \frac{3}{4}x - \frac{47}{2}$ E. NOTA

Answer D. Finding the center as $(-1, -18)$ shows the center as 3 units to the right and 4 units below point A. We can use the same translation to find point D as $(2, -22)$. The tangent would have the same slope as the given tangent line, $\frac{3}{4}$, therefore the equation is $y = \frac{3}{4}x - \frac{47}{2}$

19. Find the coordinates of point E such that minor arc EA is congruent to minor arc DB.

A. $(-5, -21)$ B. $(-5, -15)$ C. $(-4, -22)$ D. $(-4, -6)$ E. NOTA

Answer C. Since congruent chords intercept congruent arcs, we simply need to make chords EA and DB congruent. This gives us $(-4, -22)$ as E.

For problems 20-23 use the following: Points A,B, D, E, and F are on circle C in clockwise order such that DF is a diameter. Chords BF and EF are congruent, m<EDF = 12x - 12, m<FDA = 7x + 3, the measure of the minor arc FB is 18x + 12. If P is the point of intersection between CD and BE, then PC = 9.

20. Solve for x.

A. 2 B. 3 C. 4 D. 5 E. NOTA

Answer E. Since BF and EF are congruent, m<EDF is half the measure of arc FB, giving us the equation: $12x - 12 = 9x + 6$ which yields $x = 6$.

21. Find m<DFE.

A. 30 B. 45 C. 60 D. 90 E. NOTA

Answer A. Since angle EDF and DFE must be complimentary because DF is the diameter. Since m<EDF = 60, m<DFE = 30.

22. Find the length of arc AFE.

A. 9π B. 15π C. $7\pi\sqrt{3}$ D. 21π E. NOTA

Answer D. Since m<DFE = 30, the measure of minor arc DE is 60, which is equal to m<DCE. Because of the congruent chords we know CD is perpendicular to BE. This makes triangle CPE and 30-60-90 triangle with a hypotenuse of 18 (which is also the radius of our circle). Since m<FDA = 45, the measure of arc AF is 90. Since the m<EDF=60, the measure of arc FE is 120. This makes the measure of arc AFE 210, taking up $\frac{7}{12}$ of the circle. The circumference is 36π, therefore, the length of the arc is 21π

23. Find the length of chord AF.

A. 9 B. $6\sqrt{6}$ C. 18 D. $18\sqrt{2}$ E. NOTA

Answer D. Since m<ACF=90, triangle ACF is an isosceles right triangle with legs of length 18 (the radius found above). Thus, $AF = 18\sqrt{2}$

24. A quadrilateral is inscribed in a circle. If I were to choose two of the four inscribed angles at random, which of the following statements must be true about the chosen angles?

A. They are supplementary.
B. They are equal.
C. They are either supplementary or equal.
D. They are neither supplementary nor equal.
E. NOTA

Answer E. If a pair of opposite angles were chosen then they would be supplementary, but if an adjacent pair of angles were chosen, no conclusion could be drawn without more conditions on the quadrilateral.

25. A right circular cone has a volume of $144\pi\ in.^3$ and a height that is twice its radius. A plane parallel to the base intersects the cone such that the distance between the base and the plane is half that between the plane and the vertex. What is the circumference of the circle formed by this intersection?

A. 4π B. $\frac{8\sqrt[3]{9}}{3}\pi$ C. 8π D. 16π E. NOTA

Answer C. With the given information we can set up the following equation: $144\pi = \frac{2}{3}\pi r^3$ which gives us $r = 6$. Since the plane intersects a third of the way up the cone, the radius of the circle of intersection will be $\frac{2}{3}$ that of the base, giving us a circumference of 8π.

26. An inscribed angle and a chord intercept the same minor arc. If the measure of the inscribed angle is 2x, which of the following is the measure of the angle made by a radius drawn to an endpoint of the chord and the chord itself?

A. $90 - 2x$ B. $\frac{180-}{2}$ C. $180 - x$ D. $180 - 2x$ E. NOTA

Answer A. The measure of the intercepted arc is 4x which is also the measure of the central angle (which is the vertex angle in the isosceles triangle formed by the chord and two radii drawn to the endpoints). The sum of the base angles would then be $180 - 4x$ so to find the measure of one, we divide by two.

27. A spiral is made taking the end of a semicircle 1 cm in radius and joining it to the end of a semicircle 2 cm in radius and then joining a semicircle 3 cm in radius to the other end of the semicircle with radius two and so on. The semicircles are joined in such a way that the radius of each smaller semicircle lies entirely on the radius of the larger semicircle. If this pattern continued, what is the least number of semicircles needed for the spiral to have a total length of over $10{,}000\pi$ cm?

A. 138 B. 139 C. 140 D. 141 E. NOTA

Answer D. To find the length of the spiral we simply add the lengths of all the semicircles that comprise it. These form an arithmetic series, allowing us to solve for the number of semicircles, n, by using the equation $n(n+1) = 20{,}000$. After solving and rounding we get 141

28. A circle of radius 1 has a second circle internally tangent to it and passing through its center. The second circle also has a third circle internally tangent to it and passing through the center of the second circle. The pattern continues infinitely. If the area between the n^{th} and the $(n+1)^{th}$ circles are shaded for all positive even integers n, what is the total shaded area?

A. $\frac{\pi}{5}$ B. $\frac{\pi}{4}$ C. $\frac{\pi}{3}$ D. π E. NOTA

Answer A. The areas of the circles generate the geometric series $\pi\left(\frac{1}{4} - \frac{1}{16} + \frac{1}{64} - \cdots\right)$, whose sum is given by the expression $\pi\frac{\frac{1}{4}}{1+\frac{1}{4}} = \frac{\pi}{5}$

29. A sphere is inscribed in a right circular cylinder. What is the ratio of the volume of the sphere to the volume of the cylinder?

A. 1:2 B. 2:3 C. 3:5 D. 5:9 E. NOTA

Answer B. The height of the cylinder must be 2r, therefore the volume of the cylinder in terms of r is $2\pi r^3$. This makes the ratio 2:3

30. What would a math test be without logs? Right cylindrical logs of radius r are stacked on the ground in rows on top of each other such that each log on the second row is tangent to two logs on the first row. In terms of r, how high is the top of the second row above the ground?

A. $\sqrt{3}r$ B. $3r$ C. $(2+\sqrt{3})r$ D. $4r$ E. NOTA

Answer C. Taking a set of three logs (two on the bottom and one on the top between them) we can create an equilateral triangle by drawing radii connecting all three centers. This triangle has a side length of 2r and a height of $r\sqrt{3}$. Its base lies r above the ground and its top is r below the top of the second row. This gives a total distance of $\left(2+\sqrt{3}\right)r$.

Answers
1. D
2. E
3. C
4. C
5. C
6. A
7. C
8. B
9. D
10. D
11. D
12. A
13. B
14. B
15. A
16. B
17. A
18. D
19. C
20. E
21. A
22. D
23. D
24. E
25. C
26. A
27. D
28. A
29. B
30. C