For the following questions, select E, NOTA if none of the above answers is correct.
Good luck and have fun!

1. The solutions to the equation $x^2 - x - 2019$ can be written in the form $\frac{A \pm B\sqrt{C}}{D}$ for positive integers A, B, C, D. If $\gcd(A, B, D) = 1$ and C is not divisible by the square of any prime, find $A + B + C + D$.
(A) 8079 (B) 8080 (C) 8081 (D) 8082 (E) NOTA

2. Let $g(x) = (x-2)^2 - (x-3)^2 + (x-4)^2 - (x-5)^2 \ldots + 2^2 - 1^2$, where x is a positive even integer. Find the value of x such that $g(x) = 105$.
(A) 10 (B) 12 (C) 14 (D) 16 (E) NOTA

3. In precalculus, you will learn that the area of a triangle can be computed using the formula $\frac{ab\,s}{2}$, where a, b are sides of the triangle and C is the included angle between the two sides. With this in mind, calculate $\sin(\angle BAC)$ in ΔABC, where $AB = 5, BC = 6, AC = 7$.
(A) $\frac{\sqrt{6}}{5}$ (B) $\frac{12\sqrt{6}}{35}$ (C) $\frac{\sqrt{6}}{7}$ (D) $\frac{2\sqrt{6}}{5}$ (E) NOTA

4. Steve has a passion for seals. He has so great a passion that he decides to have a pet seal in his home. To protect the seal, he needs to fence in an area where his seal can roam. If Steve has 80 feet of fencing available, what is the biggest area, in square feet, that he can enclose for his seal?
(A) $\frac{1600}{\pi}$ (B) $\frac{1600\sqrt{3}}{9}$ (C) 400 (D) $64\sqrt{25 + 10\sqrt{5}}$ (E) NOTA

5. Let all the solutions to the equation
$$|x^2 - 3x + 2| + |y^2 - 5y + 6| = 0$$
be $(x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)$.
Compute $\sum_{j=1}^{n} x_j y_j$.
(A) 0 (B) 12 (C) 15 (D) 27 (E) NOTA

6. Given that $x + y = 3$, let the minimum value of $x^2 - 2x + y^2 - 4y + 7$ be A. Find the greatest integer less than or equal to A.
(A) 1 (B) 2 (C) 3 (D) 4 (E) NOTA

7. Viraj is on spring break! As many competitions are getting close, Viraj's spring break is productive, to say the least. All he does every day is do math, sleep, and code. The number of hours that Viraj does math is directly proportional to his AIMEII score, and inversely proportional to the product of the numbers of hours that he sleeps and codes. When he does math for 2 hours a day and codes for 7 hours a day, he gets an AIMEII score of 4. Viraj's AIMEII score is K when he does 7 hours of math a day and sleeps for 9 hours a day. Find the integer closest to K.
(A) 7 (B) 8 (C) 9 (D) 10 (E) NOTA

8. A polynomial $p(x)$ satisfies the equation:
$$p(x)^2 - (x^2 + x - 2)p(x) + (x^3 - 3x^2 + x - 3) = 0$$
Let all possible values of $p(-1000)$ be $y_1, y_2, y_3, \dots y_n$, such that $y_1 > y_2 > \cdots > y_n$. Compute $S(|y_1| + |y_2| + \cdots |y_n|)$, where $S(X)$ is the sum of digits of a number X.

(A) 6 (B) 7 (C) 8 (D) 9 (E) NOTA

9. If a, b, c satisfy $-4(a + b + c) = (abc) = 4(ab + bc + ac)$ and $abc = -16$, then calculate the sum of all distinct possible values of $a^3 + b^3 + c^3$.
(A) 56 (B) 64 (C) 72 (D) 80 (E) NOTA

10. Find the number of real solutions to the equation:
$$\sqrt{3 - x^2} = \sqrt{9 - 5x}$$
(A) 1 (B) 2 (C) 3 (D) 4 (E) NOTA

11. Claire's Violin shop sells violins of all shapes and sizes! Arjun has played the guitar for many years, and he wants a more classical noise. He decides to buy a violin. Arjun plans to shop on Black Friday. As he enters the store, he sees a banner that reads: "Solve this math problem, and you will get a 10% off discount on any violin you choose." As violins cost a bit of money, Arjun immediately jumps on the deal, and to his joy, he realizes he knows how to solve the problem! The problem reads:
"Find the volume of a triangular prism with a height of 3 and a triangular base with 3 sides of length 8." Find the correct answer that Arjun gave Claire.
(A) $16\sqrt{3}$ (B) $48\sqrt{3}$ (C) 32 (D) 96 (E) NOTA

12. A cubic with leading coefficient 2 has the property that $f(1) = 1, f(2) = 8$, and $f(3) = 27$. Compute $f(4)$.
(A)0 (B) 64 (C) 70 (D) 76 (E) NOTA

13. If $\begin{vmatrix} f & z & \& \\ r & e & l \\ a & r & u \end{vmatrix} = 3$, calculate the sum of the digits of $\begin{vmatrix} 4f & 4z & 4\& \\ 4r & 4e & 4l \\ 4a & 4r & 4u \end{vmatrix}$.

(Note: the entries in the matrix represent distinct numbers.)
(A) Not enough info (B) 0 (C) 3 (D) 12 (E) NOTA

14. Calculate the number of real solutions to the equation:
$$8^x + 15^x = 17^x$$

(A) 0 (B) 1 (C) 2 (D) 3 (E) NOTA

15. Sohan has been working out recently at the gym. Let $P(x)$ be the number of pounds that Sohan can bench after x months. If

$$P(x) = -x^4 + 4x^3 - 6x^2 + 4x + 148$$

then find the maximum number of pounds Sohan can bench.

(A) 148 (B) 149 (C) 150 (D) 151 (E) NOTA

16. Calculate the number of real solutions to the equation:

$$\lfloor x^2 \rfloor - x - 1980 = 0$$

where $\lfloor a \rfloor$ is the greatest integer less than or equal to a.

(A) 1 (B) 2 (C) 4 (D) 8 (E) NOTA

For the following two questions, $g(x) = x^3 - 5x^2 + 12x + 1$, where a, b, c are the roots. You will be given expressions in terms of a, b, c. Your mission is to calculate these values and select the choice that represents the correct value.

17. $(a-2)(b-2)(c-2)$

(A) 0 (B) -11 (C) -12 (D) -13 (E) NOTA

18. $\frac{a^2+b^2}{c^2} + \frac{a^2+c^2}{b^2} + \frac{c^2+b^2}{a^2}$

(A) 131 (B) 134 (C) 151 (D) 154 (E) NOTA

19. Find the sum of all the values of n for which the given equation will have exactly one real root: $x^2 + nx + 3 - 2n = 0$

(A) -8 (B) -1 (C) 2 (D) 6 (E) NOTA

20. Find the minimum value of $x + \frac{9}{x}$.

(A) 1 (B) 3 (C) 6 (D) 10 (E) NOTA

21. Let $j(x) = \frac{x^2-2019x+2019}{x-1}$. Compute the unique integer r such that $\lfloor j(r) \rfloor = 2019$.
Note: $\lfloor a \rfloor$ is the greatest integer less than or equal to a.
(A) 4035 (B) 4036 (C) 4037 (D) 4038 (E) NOTA

22. If the positive real solution to $x\lfloor x \rfloor = 84$ can be represented as $\frac{a}{b}$, where a and b are relatively prime positive integers, then compute $a+b$.
Note: $\lfloor a \rfloor$ represents the greatest integer less than or equal to a.
(A) 25 (B) 27 (C) 29 (D) 31 (E) NOTA

23. We define a Gaussian integer to be a complex number of the form $a + bi$, where a, b are integers. Additionally, the magnitude of such a number is defined to be $\sqrt{a^2+b^2}$. Find the number of distinct Gaussian integers with magnitude 100.
(A) 8 (B) 12 (C) 16 (D) 20 (E) NOTA

24. Given that the polynomial $y^4 - 10y^3 + 29y^2 - 22y + 4$ has 4 positive real roots, find the sum of the solutions to the polynomial:
$$e^{4x} - 10e^{3x} + 29e^{2x} - 22e^x + 4 = 0$$
(A) ln4 (B) ln10 (C) 4 (D) 10 (E) NOTA

25. Find the unique positive integer x such that $\frac{4^x-13}{2^{x-1}+1} = 2^{x+1} - 2^{x-2} - 2^0$.
(A) 2 (B) 3 (C) 4 (D) 5 (E) NOTA

26. Let $m = (x+y-z)(x-y+z)(-x+y+z)(x+y+z)$. Compute $\lfloor \sqrt{m} \rfloor$ when $x = \sqrt{5}, y = 2\sqrt{2}, z = \sqrt{13}$. ($\lfloor a \rfloor$ is the greatest integer less than or equal to a.)
(A) 10 (B) 12 (C) 14 (D) 16 (E) NOTA

27. The quadratic $x^2 + ax + b$ has at least one real root, where a, b are integers and $0 \leq a, b \leq 10$. Find the number of distinct ordered pairs (a, b) for which this is true.
(A) 69 (B) 71 (C) 73 (D) 75 (E) NOTA

28. Find the positive difference between the harmonic mean of 40 and 60 and the arithmetic mean of 40 and 60.

(A) 0 (B) 2 (C)4 (D) 6 (E) NOTA

29. Classify the following system of equations:

$$6x + 7y = 5$$
$$24x + 28y = 20$$

I. Independent
II. Dependent
III. Consistent
IV. Inconsistent

(A) I, III (B) II, IV (C) I, IV (D) II, III (E) NOTA

30. Compute the number of complex roots (counted with multiplicity):

$$x^9 - 234x^8 + x^7 - x^6 + 222x^5 + 432x^4 + 2019$$

(A) 9 (B) 234 (C) 432 (D) 2019 (E) NOTA