Theta Individual National Convention 2019

1. For how many ordered pairs of positive integers (x, y) is $x + 2y = 100$?

a. 33 b. 49 c. 50 d. 99 e. NOTA

2. The ratio of the supplements of two angles is 9:8 and the ratio of their complements is 3:2. Find the sum of the degree measures of the original angles.

a. 54 b. 90 c. 105 d. 130 e. NOTA

3. There are some integers k less than 10 for which $\sqrt{k+3+2+1}$ has an integral value. What is the sum of these numbers?

a. -10 b. -6 c. -4 d. 6 e. NOTA

4. Find the length of the tangent segment from (3,7) to the circle $x^2 + y^2 + 8x - 2y - 8 = 0$.

a. $2\sqrt{10}$ b. $2\sqrt{15}$ c. $\sqrt{65}$ d. $\sqrt{85}$ e. NOTA

5. The average value of all the coins in Mu-Lu's piggy bank is 20 cents. If he had one more quarter, the average value would be 21 cents. How many dimes does he have in his piggy bank, if his piggy bank contains only pennies, nickels, dimes, and quarters?

a. 1 b. 2 c. 3 d. 4 e. NOTA

6. Mr. Lu wrote down 2 numbers. One number is two times the other, and their sum is 6 more than twice the smaller number. What is the least possible number Mr. Lu wrote down?

a. 2 b. 4 c. 6 d. 12 e. NOTA

7. Mu Lu tosses a fair coin, after which Wiggie throws a fair die. They continue this pattern until Mu Lu gets a tail and wins or Wiggie throws a 5 and wins. What is the probability that Mu Lu wins?

a. $\frac{1}{3}$ b. $\frac{6}{11}$ c. $\frac{5}{7}$ d. $\frac{6}{7}$ e. NOTA

8. Not counting just 2019 itself as "1 consecutive integer", the least number of consecutive integers whose sum is 2019 is 2, since 1009 + 1010 = 2019. What is the largest number of consecutive integers whose sum is 2019?

a. 2 b. 65 c. 2019 d. 4038 e. NOTA

9. In the expansion of $\left(-x^3+3x^2-4x+7\right)^{2019}$, the term with the highest degree is $-x^{6057}$. What is the coefficient of the term with the second highest degree?

a. -6057 b. -2019 c. 2019 d. 6057 e. NOTA

10. Goldbach's conjecture states that any even integer greater than 2 can be written as the sum of two primes. For such representations of the even integer 126, what is the largest possible difference between the two primes?

a. 88 b. 92 c. 100 d. 112 e. NOTA

11. Mr. Lu buys a large number of math books at 3 for $100 and an equal number at 5 for $200. To break even, he must sell all at:

a. 8 for $300 b. 3 for $110 c. 5 for $180 d. 11 for $400 e. NOTA

12. Each of the valves M, A, and T, when open, releases water into a tank at its own constant rate. With all three valves open, the tank fills in 1 hour, with only valves M and T open it takes 1.5 hours, and with only valves A and T open it takes 2 hours. The number of hours required with only valves M and A open is:

a. 1.1 b. 1.2 c. 1.25 d. 1.75 e. NOTA

13. You are given triangle ZLU with point F on side $\overline{LU}$ and point W on side $\overline{ZU}$. If $\overline{ZL} = \overline{ZU}$, angle LZF measures 30 degrees, and $\overline{ZW} = \overline{ZF}$, what is the degree measure of angle UFW?

a. 7.5 b. 10 c. 15 d. 20 e. NOTA

14. Find the area enclosed by the graph of $|x-2|+|y|=4$.

a. Not enclosed b. 16 c. 32 d. 64 e. NOTA

15. The pressure of wind on a sail varies jointly as the area of the sail and the square of the speed of the wind. The pressure on a square foot of sail is 1 pound when the wind speed is 16 mph. When the pressure on a square yard of sail is 36 pounds, what is the speed (in mph) of the wind?

a. $\frac{8\sqrt{3}}{3}$ b. 32 c. 96 d. $192\sqrt{3}$ e. NOTA

16. Find the sum of the x and y coordinates of the point that is equidistant from (-2,5), (6,7), and (8,5).

a. 2 b. 3 c. 4 d. 5 e. NOTA

17. Find the largest prime divisor of $2^{12} + 5^{12} - 2 \bullet 10^6$.

a. 2 b. 3 c. 19 d. 29 e. NOTA

18. List the numbers of imaginary and real solutions, as an ordered pair (# imaginary, # real), that satisfy the equation $\frac{k^6 - 8}{k^2 - 2} - 12 = 0$.

a. (2,0) b. (2,2) c. (0,2) d. (4,0) e. NOTA

19. How many integer values of x **do not** satisfy the following inequality $\frac{2}{x-2} - \frac{2}{x+3} \geq -1$?

a. infinitely b. 4 c. 5 d. 6 e. NOTA

20. Find the number of positive factors of $3! \cdot 5! \cdot 7!$ that are perfect cubes.

a. 4 b. 5 c. 6 d. 14 e. NOTA

21. The number of ordered triples (Z,L,U) of positive integers that satisfy the system $\begin{matrix} ZL + LU = 44 \\ ZU + LU = 23 \end{matrix}$ is:

a. 1 b. 2 c. 3 d. 4 e. NOTA

22. One gallon of paint is needed to paint a 6-meter-high statue of Mr. Lu. How many gallons will it take to paint 540 1-meter-high statues similar to the original?

a. 15 b. 30 c. 45 d. 90 e. NOTA

23. If the sum of all but one of the angles of a convex polygon is 2019 degrees, then the number of sides of the polygon must be

a. 12 b. 14 c. 16 d. 18 e. NOTA

24. If L=10^{-2018} and U=$2 \bullet 10^{-2019}$, then what is $L - U$?

a. $8 \bullet 10^{-1}$ b. $8 \bullet 10^{-2019}$ c. $8 \bullet 10^{-2018}$ d. 5 e. NOTA

25. Two tangents are drawn to a circle from an exterior point M; they intersect the circle at points L and U. A third tangent intersects segment ML in W and segment MU in F, and intersects the circle at J. If ML = 30, then the perimeter of the triangle MWF is:

a. 60 b. 61 c. 63 d. 64 e. NOTA

26. Triangle DOK has median KR, centroid F, and altitude KL. If KL = 15 and RL = 8, find FR.

a. $\frac{\sqrt{161}}{3}$ b. $\frac{17}{3}$ c. $\frac{34}{3}$ d. 17 e. NOTA

27. Simplify: $\frac{3^{\log_9 360}}{3^{\log_{81} 100}}$

a.3 b. 6 c. 9 d. 36 e. NOTA

28. In Las Vegas, a child is born every 12 minutes and a death occurs every 40 minutes. A citizen moves out of Las Vegas every 20 minutes, and a new person moves into Las Vegas every 6 hours. How many minutes does it take for the population to increase by four people?

a.90 b. 180 c.360 d. 720 e. NOTA

29. The equation of the line having y-intercept $\frac{4}{5}$ and x-intercept $\frac{3}{4}$ can be represented in the form Zx + Ly = U, where Z, L, and U are relatively prime, and Z > 0. What is the value of Z + L + U?

a. -13 b. 13 c. 32 d. 43 e. NOTA

30. If $g(x)$ is a function that satisfies $x^3 = x \cdot g(x) + g(-x)$, what is the value of $g(2)$?

a. $\frac{8}{5}$ b. $\frac{8}{3}$ c. 4 d. 8 e. NOTA