

Theta Individual National convention 2019 Solutions

1. $y=1$ to $y=49$ so 49 ordered pairs B

$$\frac{180-y}{180-x} = \frac{9}{8} \rightarrow \frac{90-y}{90-x} = \frac{3}{2}$$

2. $9(180-x) = 8(180-y) \rightarrow 3(90-x) = 2(90-y)$ C
 $180 = 9x - 8y$
 $90 = 3x - 2y \rightarrow x = 60 \rightarrow y = 45 \rightarrow x + y = 105$

3. $3-2-5-6 = -10$ A

4. $x^2 + 8x + 16 + y^2 - 2y + 1 = 8 + 16 + 1$
 $(x+4)^2 + (y-1)^2 = 25 \rightarrow ctr(-4,1)$

tangent segment: $\sqrt{7^2 + 6^2} = \sqrt{85} \rightarrow 85 - 25 = A^2 \rightarrow A = 2\sqrt{15}$ B

5. $\frac{T}{n} = 20 \rightarrow \frac{T+25}{n+1} = 21 \rightarrow T = 20n$ E(0)
 3 quarters and one nickel, so no dimes.
 $\frac{20n+25}{n+1} = 21 \rightarrow n = 4 \rightarrow T = 80$

6. $x = 2y \rightarrow x + y = 6 + 2x \rightarrow y = -6 \rightarrow x = -12$ E(-12)

7. $\frac{1}{2} + \frac{1}{2} \cdot \frac{5}{6} \cdot \frac{1}{2} + \dots = \frac{\frac{1}{2}}{1 - \frac{5}{12}} = \frac{\frac{1}{2}}{\frac{7}{12}} = \frac{6}{7}$ D

8. This would be integers from -2018 to 2019 inclusive. That is a total of 4038 D

9. The second term is $2019 \cdot (-x^3)^{2018} \cdot (3x^2)$ D

10. 113 and 13 is the best you can do so answer is 100 C

$$11. \frac{100}{3} + 40 = \frac{220}{3} \rightarrow \frac{220}{3} \div 2 = \frac{110}{3} \quad \text{B}$$

$$12. \begin{aligned} M + A + T = 1 &\rightarrow M + T = \frac{2}{3} \rightarrow A + T = \frac{1}{2} \\ A = \frac{1}{3} &\rightarrow M = \frac{1}{2} \rightarrow M + A = \frac{5}{6} \rightarrow \frac{5}{6}t = 1 \rightarrow t = \frac{6}{5} \end{aligned} \quad \text{B}$$

13. Draw a picture. Call angles ZLF=x and ZUF=x also. Call angles ZWF and ZFW=y. Then angle of interest is y-x. Angle ZFL=150-x. So $150-x+y+y-x=180$. $2y-2x=30$. $y-x=15$ C

14. Draw the picture and you get a square with diagonal of length 8 and therefore a side length of $4\sqrt{2}$. Therefore the area is 32 C

$$15. \frac{P}{AS^2} = \frac{1}{1 \cdot 16^2} = \frac{36}{9S^2} \rightarrow S^2 = 4 \cdot 16^2 \rightarrow S = 32 \quad \text{B}$$

$$16. \begin{aligned} \frac{7-5}{6-2} &= \frac{1}{4} \rightarrow \text{midpoint} = (2, 6) \rightarrow y = -4x + 14 \\ \frac{7-5}{6-8} &= -1 \rightarrow \text{midpoint} = (7, 6) \rightarrow y = x - 1 \\ (3, 2) &\rightarrow 3 + 2 = 5 \end{aligned} \quad \text{D}$$

$$17. \begin{aligned} 5^6 - 2^6 &= (5^3 - 2^3)(5^3 + 2^3) = 117 \cdot 133 \\ 3^2 \cdot 13 \cdot 7 \cdot 19 &\rightarrow 19 \end{aligned} \quad \text{C}$$

$$18. \begin{aligned} k^4 + 2k^2 + 4 = 12 &\rightarrow k^4 + 2k^2 - 8 = 0 \\ (k^2 + 4)(k^2 - 2) &= 0 \rightarrow (2, 0) \end{aligned} \quad \text{A}$$

$$\frac{2(x+3) - 2(x-2) + (x-2)(x+3)}{(x-2)(x+3)} \geq 0$$

19. $\frac{2x+6-2x+4+x^2+x-6}{(x-2)(x+3)} \geq 0 \rightarrow \frac{x^2+x+4}{(x-2)(x+3)} \geq 0$ D
 $-3, -2, -1, 0, 1, 2 \rightarrow 6$

20. $3 \cdot 2 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 = 3^4 \cdot 2^8 \cdot 5^2 \cdot 7$ C
 $1, 2, 3, 4, 6, 12 \rightarrow 6$

$$L(Z+U) = 44 \rightarrow U(Z+L) = 23 \rightarrow Z+L = 23 \rightarrow U = 1$$

21. $44 = 1 \cdot 44, 4 \cdot 11, 2 \cdot 22 \rightarrow L = 22 \rightarrow Z+1 = 2 \rightarrow L = 2 \rightarrow Z+1 = 22$ B
 $(1, 22, 1)(21, 2, 1) \rightarrow 2$

22. $scale\ factor = \frac{1}{6} \rightarrow area = \frac{1}{36} \rightarrow \frac{540}{36} = 15$ A

23. $180 \cdot 11 = 1980 \rightarrow 180 \cdot 12 = 2160 \rightarrow n - 2 = 12 \rightarrow n = 14$ B

24. $L - U = 10^{-2019} (10 - 2) = 8(10^{-2019})$ B

25. If you draw a picture. We know tangent segments are congruent. If we call $WL=x$, then so is $WJ=x$. If we call $UF=y$ then $JF=y$. This leaves our triangle with sides $30-x$, $30-y$, and $x+y$. Add those up and you get 60 A

26. If you draw a picture you see triangle KLr is a right triangle so side KR equals 17. The centroid is where the medians meet. They meet in the ratio of 2:1 so $Fr=17/3$ B

27. $\frac{3^{\log_9 360}}{3^{\log_{81} 100}} = 3^{\log_9 360 - \log_{81} 100} = 3^{\log_9 360 - \log_9 10} = 3^{\log_9 36} = 3^{\log_3 6} = 6$ B

28. $\frac{1}{12} - \frac{1}{40} - \frac{1}{20} + \frac{1}{360} = \frac{30-9-18+1}{360} = \frac{4}{360} \rightarrow 360$ C

29. . Find slope: $\frac{\frac{4}{5}}{\frac{-3}{4}} = \frac{-16}{15}$. Put in standard form; $16x + 15y = C$. Plug in point to solve for C and

you get $16x + 15y = 12$. $15 + 16 + 12 = 43$

D

$$8 = 2g(2) + g(-2)$$

30. $-8 = -2g(-2) + g(2) \rightarrow 8 = 5g(2) \rightarrow g(2) = \frac{8}{5}$

A

Answers:

1. B

2. C
3. A
4. B
5. E(0)
6. E(-12)
7. D
8. D
9. D
10. C
11. B
12. B
13. C
14. C
15. B
16. D
17. C
18. A
19. D
20. C
21. B
22. A
23. B
24. B
25. A
26. B
27. B
28. C
29. D
30. A