All uppercase letter variables are positive integers unless otherwise stated. All fractions containing uppercase letter variables are in lowest terms. NOTA means "None of the Above."

~~~~~~~~ *Good luck, and have fun!* ~~~~~~~~

1) It is given that $1^0 = 1$. Find the value of $\log_1 1$.

(A) 0
(B) 1
(C) Both (A) and (B)
(D) Undefined
(E) NOTA

2) Find the solution to the equation $\log_2(\log_2 x + \log_4(x^2)) = 2019$.

(A) 2^{2019}
(B) $2^{2^{2018}}$
(C) $2^{2^{2019}-1}$
(D) $2^{2^{2019}}$
(E) NOTA

3) Evaluate: $\log_{0.03125} 0.25$

(A) 0.4
(B) 0.5
(C) 2
(D) 2.5
(E) NOTA

4) Find the number of positive integer factors of 72^3.

(A) 36
(B) 48
(C) 56
(D) 70
(E) NOTA

5) Given that $\log_2 3 = \alpha$, $\log_2 5 = \beta$, and $\log_2 7 = \gamma$, find the value of $\log_{84} 300$.

(A) $\frac{1+2\beta}{2+\gamma}$
(B) $\frac{2+\alpha+\beta}{1+2\alpha+\gamma}$
(C) $\frac{2+\alpha+2\beta}{2+\alpha+\gamma}$
(D) $\frac{1+2\alpha+\beta}{2+\alpha+2\gamma}$
(E) NOTA

6) Find the domain of the function $f(x) = 2019^{x^{-2}} \log_{\sqrt{18-3x-x^2}}(x^2 + 2x - 8)$.

(A) (-6,-4)
(B) (-6,-4) ∪ $(2,3)$
(C) $(-4,0) \cup (0,3)$
(D) $(2, \infty)$
(E) NOTA

7) Evaluate: $(1+i)^{2019}(1-i)$, where $i = \sqrt{-1}$.

(A) 2^{1010}
(B) $2^{1010}i$
(C) -2^{1010}
(D) $-2^{1010}i$
(E) NOTA

8) Find the product of all values of x such that the matrix $\begin{bmatrix} 2 & 3 & -\ln x \\ 5\ln x & 1 & 0 \\ -2 & -4 & 3\ln x \end{bmatrix}$ does not have an inverse.

(A) $e^{4/25}$
(B) $e^{8/25}$
(C) $e^{2/5}$
(D) $e^{8/15}$
(E) NOTA
~~~~~~~~

9) Given that k is a positive integer greater than 2019, find the number of complex values of x such that the equation $x^k = 2019$ is satisfied.

(A) 1
(B) 2
(C) 1 if k is odd, 2 if k is even
(D) k
(E) NOTA

10) For how many integers $n \leq 200$ is the expression $\sqrt{n + \sqrt{n + \sqrt{n + \cdots}}}$ a positive integer?

(A) 14
(B) 15
(C) 28
(D) 29
(E) NOTA

11) Find the product of the solutions to $(\ln x)^2 + 8\ln x + 4 = 0$.

(A) e^{-8}
(B) e^{-4}
(C) 4
(D) e^4
(E) NOTA

12) Find the units digit of $2^{2019} + 0^{2019} + 1^{2019} + 9^{2019} + 2019^2 + 2019^0 + 2019^1 + 2019^9$.

(A) 6
(B) 7
(C) 8
(D) 9
(E) NOTA

13) Given that $a > b > 1$, find a^k, where $k = \frac{\log_b(\log_b a)}{\log_b a}$.

(A) $\log_b(\log_a b)$
(B) $\log_b(\log_b a)$
(C) $\log_a b$
(D) $\log_b a$
(E) NOTA

14) Evaluate: $\log_2 2^{3^4}$

(A) 12
(B) 81
(C) 2^{12}
(D) 2^{81}
(E) NOTA

15) The number 12^8 is equal to $\overline{429X81696}$, where $\overline{X}$ is some digit. Find $\overline{X}$.

(A) 0
(B) 4
(C) 8
(D) 9
(E) NOTA

16) Given the following equation, find $RU + SV + TW$.

$$\frac{x^{2a+b+2}y^{3a+4b-5}}{y^{-4a+6b+1}x^{3a-3b-2}} \div \frac{x^{4a+2b}y^{5a-2b+4}}{x^{-2a-6b-1}y^{-a+b+3}} = x^{Ra+Sb+T} \cdot y^{Ua+Vb+W}$$

(A) -130
(B) -32
(C) -15
(D) 25
(E) NOTA

17) Find the remainder when 7^{1350} is divided by 2019.

(A) 382
(B) 547
(C) 655
(D) 1810
(E) NOTA

18) Given that $\log_{16} 27 \cdot \log_{49} 512 \cdot \log_{625} 343 \cdot \log_{729} 15625 = \frac{H}{K}$, find $H + K$. (If needed, check the directions for restrictions on H and K.)

(A) 43 (B) 59 (C) 97 (D) 113 (E) NOTA

19) Trevor has $1337. He invests it all in an account that pays 6% per year (compounded continuously). Which of the following expressions represents how many dollars Trevor has after t years?

(A) $1337e^{0.06}$

(B) $1337 + 1337e^{0.06}$

(C) $1337(1.06)^t$

(D) $1337e^{1.06}$

(E) NOTA

20) Find the sum of the distinct real solutions to $(x^2 - 7x + 11)^{x^2+4x-45} = 1$.

(A) -2 (B) 1 (C) 3 (D) 5 (E) NOTA

21) For a function $f(x) = ab^x$, it is given that $f(3) = 6$ and $f(6) = 3$. Find b^{-a}.

(A) 4 (B) 8 (C) 16 (D) 32 (E) NOTA

22) Which of the following gives the number of digits in the decimal expansion of $5^{500}2^{1000}$? (It may be helpful to know that $\log_{10} 5$ to 3 significant figures is 0.699.)

(A) 648 (B) 649 (C) 650 (D) 651 (E) NOTA

23) Given the following definition of $f(x)$, in which of the following ranges is $f(2019)$?

$$f(x) = \sum_{n=1}^{x} \lceil \log_{12} n \rceil$$

(A) $[6100,6199]$ (B) $[6200,6299]$ (C) $[6300,6399]$ (D) $[6400,6499]$ (E) NOTA

24) Convert $\frac{2}{3}$ to binary.

(A) $0.\overline{100}_2$ (B) $0.\overline{10}_2$ (C) $0.\overline{101}_2$ (D) $0.10\overline{1}_2$ (E) NOTA

25) If the solution to $9^{3x+7} = 243^{2x-1}$ is $\frac{A}{B}$, find $A + B$.

(A) 10 (B) 18 (C) 23 (D) 25 (E) NOTA

26) Solve for x: $\sqrt[x]{e} > 1$

(A) $x \in (-\infty, 0)$
(B) $x \in (0,1)$
(C) $x \in (0, \infty)$
(D) $x \in (1, \infty)$
(E) NOTA

27) Eridan and Terezi are taking turns flipping an unfair coin, which has been weighted to come up heads 1/3 of the time. If Eridan flips first, find the probability that he flips the first head.

(A) $1/3$
(B) $1/2$
(C) $4/7$
(D) $3/5$
(E) NOTA

28) The general solution to the recursion $x_{n+2} = 9x_{n+1} - 20x_n$, given initial values of $x_0 = 10$ and $x_1 = 47$, is $x_n = A_1B_1^n + A_2B_2^n$. Find $A_1 + A_2 + B_1 + B_2$.

(A) 13
(B) 17
(C) 19
(D) 23
(E) NOTA

29) Evaluate 2019^n, where n is equal to the following expression:

$$\prod_{k=2}^{2019} \frac{1}{\log_k 2019}$$

(A) $\sqrt[2019!]{2019}$
(B) $\sqrt[2019]{2019}$
(C) $\sqrt[2019]{2019!}$
(D) $2019!$
(E) NOTA

30) In linear algebra, a **vector space** is defined as a nonempty set V, on which two operations (vector addition, denoted here as $\oplus$, and scalar multiplication) are defined, subject to the ten axioms shown. The axioms must hold for *all* vectors $\boldsymbol{u}, \boldsymbol{v}, \boldsymbol{w} \in V$ and *all* scalars $c, d \in \mathbb{R}$.

(1) $\boldsymbol{u} \oplus \boldsymbol{v} \in V$.
(2) $\boldsymbol{u} \oplus \boldsymbol{v} = \boldsymbol{v} \oplus \boldsymbol{u}$.
(3) $(\boldsymbol{u} \oplus \boldsymbol{v}) \oplus \boldsymbol{w} = \boldsymbol{u} \oplus (\boldsymbol{v} \oplus \boldsymbol{w})$.
(4) There exists an identity vector **0** such that $\boldsymbol{u} \oplus \boldsymbol{0} = \boldsymbol{u}$.
(5) For each $\boldsymbol{u} \in V$, there exists a vector $-\boldsymbol{u} \in V$ such that $\boldsymbol{u} \oplus (-\boldsymbol{u}) = \boldsymbol{0}$.
(6) $c\boldsymbol{u} \in V$.
(7) $c(\boldsymbol{u} \oplus \boldsymbol{v}) = c\boldsymbol{u} \oplus c\boldsymbol{v}$
(8) $(c + d)\boldsymbol{u} = c\boldsymbol{u} \oplus d\boldsymbol{u}$
(9) $(cd)\boldsymbol{u} = c(d\boldsymbol{u})$
(10) $1\boldsymbol{u} = \boldsymbol{u}$

The operations in a proposed 1-dimensional vector space $V = \mathbb{R}^+$ are given as $\boldsymbol{u} \oplus \boldsymbol{v} = \boldsymbol{uv}$ and $c\boldsymbol{u} = \boldsymbol{u}^c$. For example, $\boldsymbol{2} \oplus \boldsymbol{3} = \boldsymbol{6}$ and $3(\boldsymbol{2}) = \boldsymbol{8}$. How many of the axioms fail for this potential vector space?

(A) 0
(B) 1
(C) 2
(D) 3
(E) NOTA