Suppose a and b are positive integers that satisfy the following:

$$a! \equiv 0 \pmod{216}$$

$$b! \equiv 0 \pmod{1000}$$

Find the smallest possible value of $a + b$.

#0 Theta Bowl
MAΘ National Convention 2019

Suppose a and b are positive integers that satisfy the following:

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Find the smallest possible value of $a + b$.

Let A be the third smallest natural number with exactly 15 positive integral factors.

Let B be the number of ordered triples (M, A, θ) of positive integers that satisfy $(M^A)^\theta = 64$.

A right triangle has one leg of length $\sqrt[4]{5}$ and a hypotenuse of length $\sqrt[4]{11+2\sqrt{30}}$. The length of the other leg is $\sqrt[4]{C}$.

Find $A + B + C$.

#1 Theta Bowl
MAΘ National Convention 2019

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Find $A + B + C$.

#2 Theta Bowl
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Let $a, b, c,$ and d be real numbers.

Suppose $a \neq 1$ and $3a^4 - 2a^2 - 1 = 0$.

Suppose $b \neq 1$ and $3b^{1/4} - 2b^{1/2} - 1 = 0$.

Suppose the equation $x^2 + 2\sqrt{2}x + c = 0$ has d distinct real solutions for x.

Suppose the equation $y^2 + 2\sqrt{2}y + d = 0$ has c distinct real solutions for y.

Find $abcd$.

#2 Theta Bowl
MAΘ National Convention 2019

Let $a, b, c,$ and d be real numbers.

Suppose $a \neq 1$ and $3a^4 - 2a^2 - 1 = 0$.

Suppose $b \neq 1$ and $3b^{1/4} - 2b^{1/2} - 1 = 0$.

Suppose the equation $x^2 + 2\sqrt{2}x + c = 0$ has d distinct real solutions for x.

Suppose the equation $y^2 + 2\sqrt{2}y + d = 0$ has c distinct real solutions for y.

Find $abcd$.

Suppose $x^4 + 4$ factors over $\mathbb{Z}$ as $(x^2 + ax + b)(x^2 + cx + d)$.

Suppose $x^4 + x^3 + x - 1$ factors over $\mathbb{Z}$ as $(x^2 + ex + f)(x^2 + gx + h)$.

Find $a + b + c + d + e + f + g + h$.

#3 Theta Bowl
MAΘ National Convention 2019

Suppose $x^4 + 4$ factors over $\mathbb{Z}$ as $(x^2 + ax + b)(x^2 + cx + d)$.

Suppose $x^4 + x^3 + x - 1$ factors over $\mathbb{Z}$ as $(x^2 + ex + f)(x^2 + gx + h)$.

Find $a + b + c + d + e + f + g + h$.

Going at 30 mph, Alice would arrive one minute early. Going at 20 mph, she would arrive one minute late. Going at A mph, she would arrive exactly on time.

Bob starts with 100 mL of a 99% acid solution. He magically extracts B mL of pure acid, leaving behind a solution that is only 98% acid.

Find $A + B$.

#4 Theta Bowl
MAΘ National Convention 2019

Going at 30 mph, Alice would arrive one minute early. Going at 20 mph, she would arrive one minute late. Going at A mph, she would arrive exactly on time.

Bob starts with 100 mL of a 99% acid solution. He magically extracts B mL of pure acid, leaving behind a solution that is only 98% acid.

Find $A + B$.

Suppose x and y are rational numbers that satisfy the following:

$$15x^3 + 17x^2 - 11x - 12 = 0$$

$$15y^3 - 11y^2 - 3y - 12 = 0$$

Find xy.

#5 Theta Bowl
MAΘ National Convention 2019

Suppose x and y are rational numbers that satisfy the following:

$$15x^3 + 17x^2 - 11x - 12 = 0$$

$$15y^3 - 11y^2 - 3y - 12 = 0$$

Find xy.

Let A be the sum of the distinct real solutions for x:

$$(x^2 + 5x + 5)^{x+5} = 1$$

Let B be the sum of the distinct real solutions for y:

$$\log(2 + y) + 2\log y + \log(2 - y) = 0$$

Find $A^2 + B^2$.

#6 Theta Bowl
MAΘ National Convention 2019

Let A be the sum of the distinct real solutions for x:

$$(x^2 + 5x + 5)^{x+5} = 1$$

Let B be the sum of the distinct real solutions for y:

$$\log(2 + y) + 2\log y + \log(2 - y) = 0$$

Find $A^2 + B^2$.

Circles C_1, C_2, C_3, and C_4 have diameters a, b, c, and d, respectively.

C_1 is circumscribed about a triangle with side lengths 3, 4, and 5.

C_2 is circumscribed about a parallelogram with side lengths 1, 2, 1, and 2.

C_3 is circumscribed about a kite with side lengths 1, 1, 2, and 2.

C_4 is circumscribed about a regular hexagon whose sides each have length 1.

Find $abcd$.

#7 Theta Bowl
MAΘ National Convention 2019

Circles C_1, C_2, C_3, and C_4 have diameters a, b, c, and d, respectively.

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C_4 is circumscribed about a regular hexagon whose sides each have length 1.

Find $abcd$.

Let a and b be positive real numbers.

Suppose $a = 3 + \sqrt{a + \sqrt{a + \sqrt{a + \cdots}}}$.

Suppose $b = 3 - \sqrt{b + \sqrt{b + \sqrt{b + \cdots}}}$.

Find $a + b$.

#8 Theta Bowl
MAΘ National Convention 2019

Let a and b be positive real numbers.

Suppose $a = 3 + \sqrt{a + \sqrt{a + \sqrt{a + \cdots}}}$.

Suppose $b = 3 - \sqrt{b + \sqrt{b + \sqrt{b + \cdots}}}$.

Find $a + b$.

Suppose f and g are functions that satisfy the following for all real numbers x:

$$2f(x) + f(1-x) = x$$

$$g\big(g(x)\big) = x^4 + 2x^3 - 2x^2 - 3x$$

$(fg)(0)$ is rational and nonzero.

Find $(fg)(0)$.

#9 Theta Bowl
MAΘ National Convention 2019

Suppose f and g are functions that satisfy the following for all real numbers x:

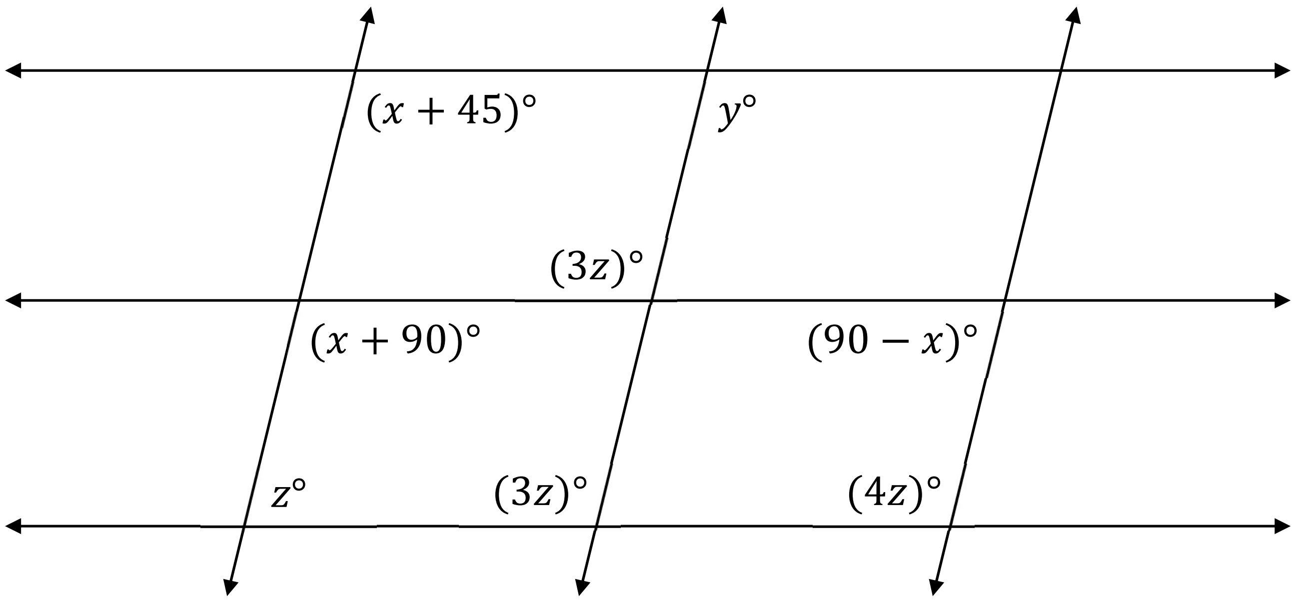
$$2f(x) + f(1-x) = x$$

$$g\big(g(x)\big) = x^4 + 2x^3 - 2x^2 - 3x$$

$(fg)(0)$ is rational and nonzero.

Find $(fg)(0)$.

Find $x + y + z$. (Diagram not drawn to scale. Don't assume lines are parallel.)



#10 Theta Bowl
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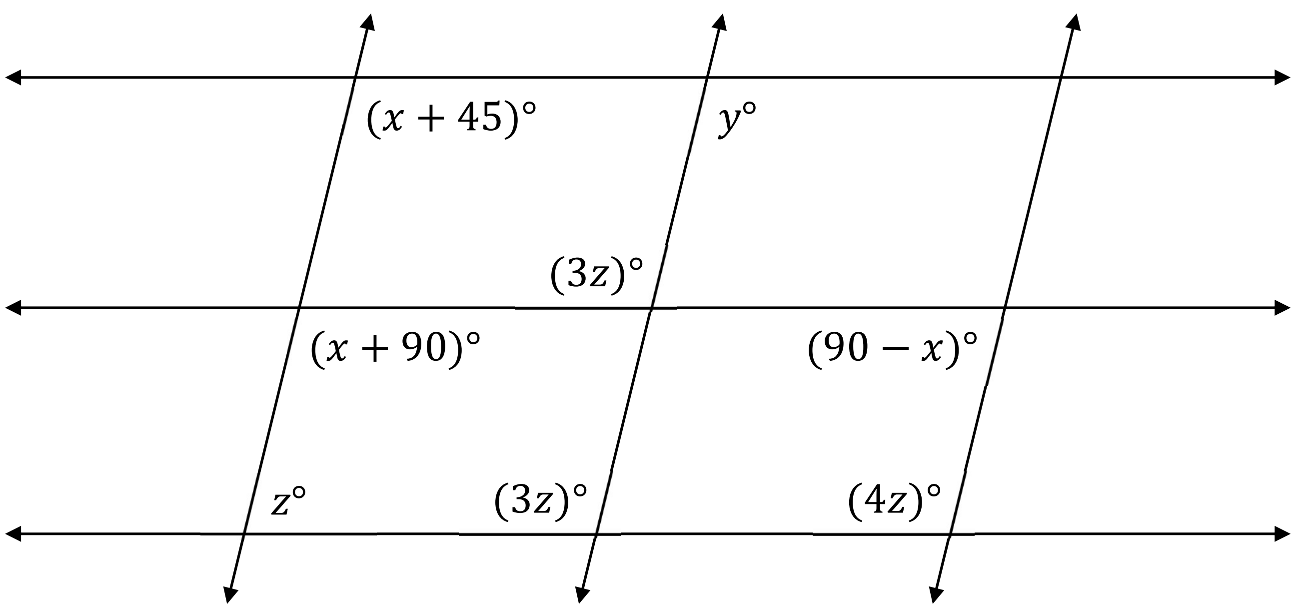
Find $x + y + z$. (Diagram not drawn to scale. Don't assume lines are parallel.)



A rectangular prism has length $2+\sqrt{3}$, width $2-\sqrt{3}$, and height 1.

Let V be its volume, let S be its surface area, and let d be the diameter of its circumscribed sphere. Finally, let $A=\frac{VS}{d^2}$.

Suppose $x, y,$ and z are positive real numbers that satisfy the following:

$$xyz = 1$$
$$yz + zx + xy = 5$$
$$x^2 + y^2 + z^2 = 15$$

Let $B = \min(x, y, z) + \max(x, y, z)$.

Find AB.

#11 Theta Bowl
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A rectangular prism has length $2+\sqrt{3}$, width $2-\sqrt{3}$, and height 1.

Let V be its volume, let S be its surface area, and let d be the diameter of its circumscribed sphere. Finally, let $A=\frac{VS}{d^2}$.

Suppose $x, y,$ and z are positive real numbers that satisfy the following:

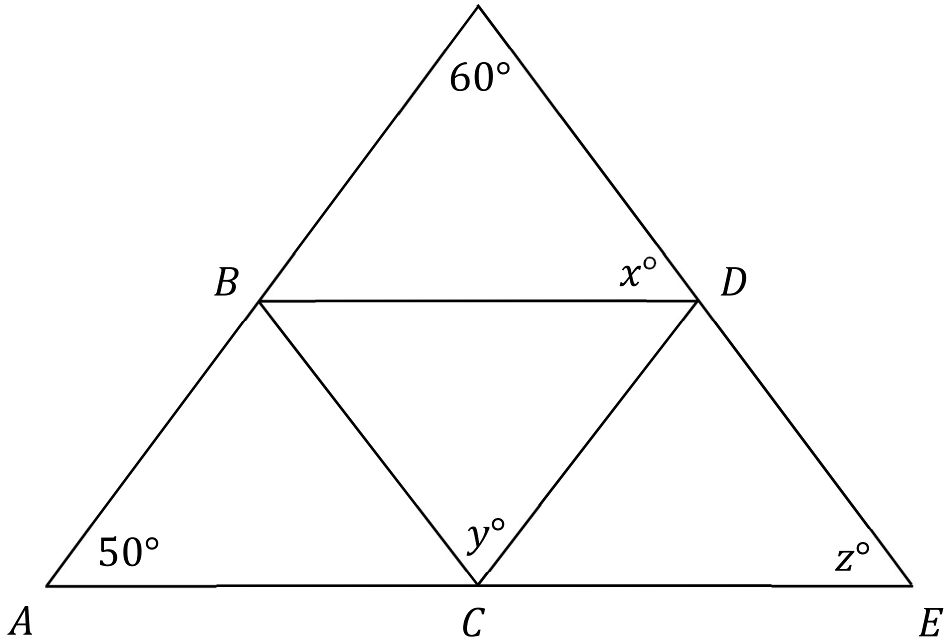
$$xyz = 1$$
$$yz + zx + xy = 5$$
$$x^2 + y^2 + z^2 = 15$$

Let $B = \min(x, y, z) + \max(x, y, z)$.

Find AB.

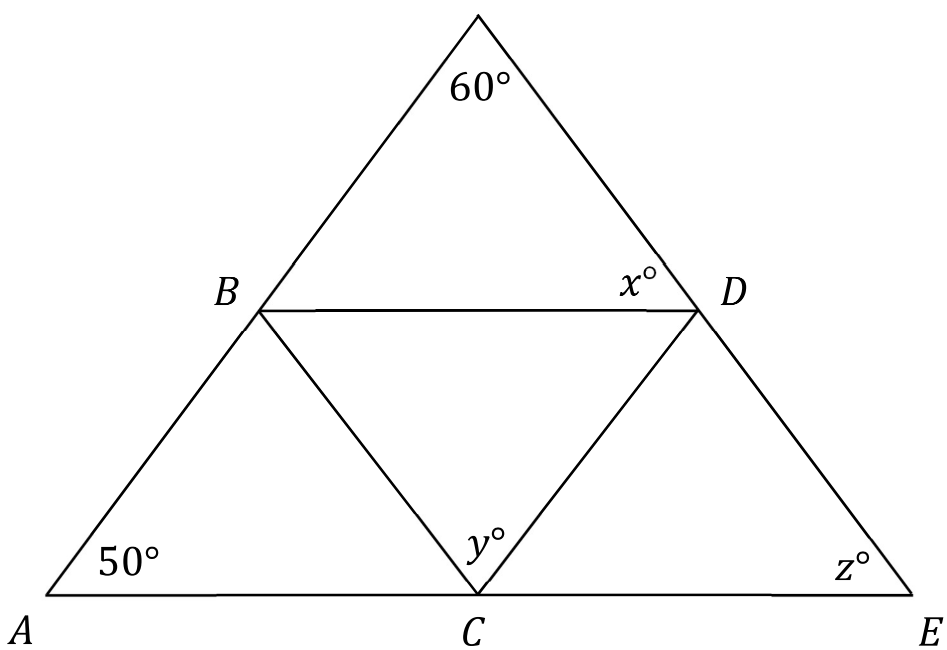
#12 Theta Bowl
MAΘ National Convention 2019

Suppose $\overline{AB} \cong \overline{BC} \cong \overline{CD} \cong \overline{DE}$. Find $x + y + z$. (Diagram not drawn to scale.)



#12 Theta Bowl
MAΘ National Convention 2019

Suppose $\overline{AB} \cong \overline{BC} \cong \overline{CD} \cong \overline{DE}$. Find $x + y + z$. (Diagram not drawn to scale.)



Let $A, B, C,$ and D be points in the xy-plane.

Suppose A is equidistant from (0, 0), (0, 1), and (2, 0).

Suppose B is equidistant from (0, 0), (2, 0), and (0, -3).

Suppose C is equidistant from (0, 0), (0, -3), and (-4, 0).

Suppose D is equidistant from (0, 0), (-4, 0), and (0, 1).

Find the area of $ABCD$.

#13 Theta Bowl
MAΘ National Convention 2019

Let $A, B, C,$ and D be points in the xy-plane.

Suppose A is equidistant from (0, 0), (0, 1), and (2, 0).

Suppose B is equidistant from (0, 0), (2, 0), and (0, -3).

Suppose C is equidistant from (0, 0), (0, -3), and (-4, 0).

Suppose D is equidistant from (0, 0), (-4, 0), and (0, 1).

Find the area of $ABCD$.

#14 Theta Bowl
MAΘ National Convention 2019

Let A be the number of distinct real solutions for x:

$$2^x x^4 + 4^x + x^2 = x^2 4^x + x^4 + 2^x$$

Let B be the product of the distinct real solutions for y:

$$|y^2 - y - 2| + |4 - y^2| + |y^2 + 3y + 2| = y^2 + 2y + 4$$

Find $A + B$.

#14 Theta Bowl
MAΘ National Convention 2019

Let A be the number of distinct real solutions for x:

$$2^x x^4 + 4^x + x^2 = x^2 4^x + x^4 + 2^x$$

Let B be the product of the distinct real solutions for y:

$$|y^2 - y - 2| + |4 - y^2| + |y^2 + 3y + 2| = y^2 + 2y + 4$$

Find $A + B$.