Mu Alpha Theta

Fall 2008

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News from the President

Welcome. I hope everyone had an eventful and enjoyable summer. Another successful Mu AlphaTheta National Convention was held in Sacramento, CA. Many thanks to Dr. Thom Morris from Berkley Prep who ran the convention smoothly and efficiently. There were some surprises that led to a great convention. Congratulations to Calvin Fukuhara from Kamehameha Schools in Honolulu, HI who won the prestigious Huneke Distinguished Sponsor Award and to Will Frazer from Buchholz High School who won the Sister Scholastica Most Committed Sponsor Award. If you did not attend, try to make the convention next summer in Knoxville, TN. The registration packet will be available online in September. The summer convention is more than competitions. There are opportunities to show your talent for scrapbooks, posters, chalk talks, and a team event based on the game "jeopardy". There are group mixers so that you can meet students from across the country and a fun day such as an amusement park, water park, or some other event showcasing the area. The fee includes rooms, meals, all activities and awards. Think seriously about attending a summer convention. Convention grants are available to schools that have not attended in the past 3 years. If you would like to apply, go to the National Convention link on our website and click the "Attend Free" tab at the top.

Last spring we gave 10 Mu Alpha Theta Scholarships each worth $4000 and the Mary Rhein Scholarship of $5000. We had many outstanding candidates and hope that you will consider applying for one of these scholarships this year. There are also 2 Andree Awards (one for a high school senior and one for a Two-Year College student) who plan to go into math education. All information is available on the Mu Alpha Theta website at www.mualphatheta.org > Scholarships and > Awards.

This year we are offering another opportunity to show your conceptual understanding and interest in a specific topic of mathematics. This will be in the form of a poster, video or Power Point presentation. The details will be on the website by October. I encourage all of the current members of Mu Alpha Theta in good standing to look over this information and submit a presentation. Many schools require a project for graduation and this would give each of you the opportunity to share your project with other students throughout the United States.

For several years Mu Alpha Theta has sponsored two free math competitions throughout the year. One is the Log 1 Contest run by the Mount Rainier High School chapter of Mu Alpha Theta in Des Moines, WA and their sponsor, Tom Tosch. This is open to Mu Alpha Theta members and consists of three tests taken right at your school. The second contest is the Rocket City Math League run by the Grissom High School Mu Alpha Theta chapter from Huntsville, AL. This contest is open to any middle school, high school or Two-Year college student and is divided into tests over specific math topics. If you are interested in participating in either or both contests talk to your Mu Alpha Theta sponsor about registering. All pertinent information is on the Mu Alpha Theta website.

The Board of Directors of Mu Alpha Theta is called the Governing Council. The four Regional Governors that serve on this Council are ready to answer any questions you might have. They are available to help students and sponsors alike. The names and e-mails of your Council members can be seen to the left. If you are not sure of your region, a regional map is available online under the link, "About Us" > Structure.

Student Delegate Officers are elected during meetings at the summer convention. Email them with your suggestions, comments and questions. They would love to connect up with you!
HAVE A GREAT YEAR.
Grace Mutz
Mu Alpha Theta President

From the National Office

Welcome to another new school year. Mu Alpha Theta continues to offer many opportunities for student members. Sponsors, please make sure to post this newsletter where all can see it or, even better, email it to your members! Besides continuing our Log 1 Contest, the Rocket City Math League, and The Actuarial Foundation's Project Math-Minds, we will offer a new **Mathematical Presentation Contest** this year. Not every student likes to take math tests for fun, so we thought giving students a chance to show off their math skills in the form of a presentation would be another great way to involve them in higher level mathematical ideas. Students may film themselves doing the presentation, make a "PowerPoint" type project or just make a poster on a topic they want to share with other math students in Mu Alpha Theta. We are still discussing the final rules and formats to be accepted, but hope to make up to $3500 in cash prizes available. Projects must be submitted electronically and will be judged by one of our Regional Governors. Details will be online soon at www.mualphatheta.org and projects will be due by February 2, 2009. We hope to allow other members to view the winning presentations in the future from links on our website. Start thinking about a topic for your presentation today!

Please take a look at the **Study Buddy** program starting up this fall. We hope this partnership with NSTEP will provide a great way for our members to tutor disadvantaged students all over the country. One of the nicest parts of the program is the ability of NSTEP to provide sponsors with the amount of service time their students are helping others. Many of our chapters require these service hours as part of a student's membership in Mu Alpha Theta. Now members can work online helping others at a time convenient to them. Check out our article on this program on page 4. An individual student may sign up or a teacher may sign up their whole club of members.

This fall Mu Alpha Theta will re-establish Chi Alpha Mu, the National Mathematics Club for middle school students. Standing for "Creative Adventures in Mathematics," the organization was created in 1966 to mirror Mu Alpha Theta. Aiming to foster a love of mathematics at a younger age, Mu Alpha Theta hopes that high school chapters will "adopt" a chapter of Chi Alpha Mu at their feeder middle schools. We will charge a one-time fee of $15 for a school to charter a chapter, but there will be no criteria for membership in the club or fees for those participating. It is our hope that students will enjoy being together to do fun math activities, maybe form a Mathcounts team, or participate in a math competition that their local Mu Alpha Theta club runs just for them. More information will be posted online as our plans solidify for the organization. Spread the word to a middle school near you!!

We hope to hold merchandise prices steady this year, even as inflation seems to be creeping up on us. One thing that has gone up is our Fed Ex shipping charges. They have continued to add higher fuel surcharges and we have raised our Two-Day delivery rate to $15 and our Overnight Delivery rate to $25 per order. We will no longer offer Next Morning Overnight delivery. The charges are just too high. Please keep this in mind when ordering since we can no longer guarantee delivery before 10 am the next morning.

Sponsors, many of you have added Associate members over the years to our database. Our system does not delete these names automatically. Please check once a year to make sure you are deleting names of students that were not eligible to become a Full member. We define an Associate member as a student in only their third or fourth semester of math and that has the math GPA to become a Full member once they begin their fifth semester. Thanks for helping to keep our database updated.

Later this year we hope to add some Student pages to our website. Students will be able to use their email address to log in and provide us with information about their interests and college plans. We hope to find mentors for students, especially once they enter college, which can help them reach their educational goals and also help them to "connect" with other students in college interested in mathematics and math competition.

Our Mathematical offering in this Newsletter is presented by Chris Cunliffe. Chris currently works in the private sector as a consulting actuary in Chicago, specializing in risk management problems for large financial institutions. He is also the President and Founder of Bobo Strategy, a company that provides useful mathematics resources. I hope you enjoy his article on Topology.

The Mu Alpha Theta National Office is staffed from 9:30 am – 6:00 pm Central time. We are closed during all national holidays and from December 24 – January 5, 2009. I do monitor emails beyond these hours.

Kay Weiss, Executive Director

Free Texas Instrument Calculators!
Once again, Mu Alpha Theta had TI 84 Plus and TI 89 Titanium calculators to give away! Our newsletter is released by email to sponsors with a current email address before being posted online or sent through the mail. Sponsors receiving calculators give them away as prizes at a competitions run by their chapter or to their outstanding members. Some chapters auction them off to raise money. Preference was given to schools that had not received free calculators in the past. Due to the overwhelming response and the lower number of calculators provided by TI this year, all were given away within 24 hours of the email release! If we don't have your current email in our database, log in and update your records so you can receive a calculator in the future.

Scholarship Application Process Simplified

Mu Alpha Theta has streamlined its application process for the Mu Alpha Theta and Mary Rhein Scholarships. Students may submit a single application for both scholarships or may use the application to apply for just one. We have added a summary information form for students to fill out and no longer require a personal resume. We feel these changes will simplify the process for both applicants and judges. The application deadline for both scholarships is **March 1, 2009**. This year, we hope to offer up to twelve $4000 Mu Alpha Theta Scholarships and one $5000 Mary Rhein Scholarship.

In April 2008, we awarded the following scholarships:

Mary Rhein Scholarship:
Laney Kuenzel, Hathaway Brown School , Shaker Heights, OH

MAT Scholarships:
David Bell of Georgetown High School in Georgetown, TX
Tong Chen of Grace King High School in Metairie, LA
Jordan Draper from Deerfield Beach High School in Deerfield Beach, FL
Hao (William) Li from Klein High School in Klein, TX
Allen Liao from Baton Rouge Magnet High School in Baton Rouge, LA
Austin Mack of North Buncombe High School in Weaverville, NC
Frederick Robinson of Holy Trinity Academy in Melbourne, FL
Nora Willett of Saint Edward's School in Vero Beach, FL
Raymond Xi of Lakota West High School in West Chester, OH
David Yakobovitch of Stoneman Douglas High School in Parkland, FL
Congratulations to all!

SUMMER GRANTS

Mu Alpha Theta offers Summer Math Study grants of up to $2000 for Full Members wishing to participate in a summer math program or a math camp at a college or university. Grants are also available for a math research project. Think about applying if you are interested in summer study or a research project! The application is online at www.mualphatheta.org > Scholarships.> Summer Grants.

This past summer, the following members were awarded grants:
Natalie Cooper, Mandeville, LA
Program: Education Program for Gifted Youth at Stanford University in CA (See Natalie's article)
Gina Pai, Columbia, MO
Program: Texas State Honors Summer Math Camp in San Marcos
Feenalie Patel, Memphis, TN
Program: Boston University High School Honors Internships program working in a research lab at Harvard University
Caleb Myers, Washington, MO
Program: Jackling Introduction to Engineering at the Missouri University of Science & Technology
Andre Kuney, West Nyack, NY
Program: Canada/USA Mathcamp at Reed College

Mu Alpha Theta Schools Win Big in M3 Challenge

In early March, over 250 teams from lower New Hampshire to Wilmington, Delaware, submitted viable solutions to the 2008 M3 Challenge problem, Energy Independence Meets the Law of Unintended Consequences. Student teams were required to address issues associated with increased corn-derived ethanol production and fuel substitution and relate these matters to dramatic and unanticipated rises in farm commodity pricing, the future of food supplies in developing nations, the effect on carbon dioxide emissions, and the cost-effectiveness of producing ethanol. Teams had 14 hours to quantify these concerns using mathematical-modeling techniques, develop and defend their models and justify their conclusions.

Two Mu Alpha Theta schools received prizes for their exceptional performance:
First Runner-up, Magna Cum Laude Team Prize of $15,000:
Manalapan High School, Manalapan, NJ – Team #72
Going Green Does Not Save Green: Corn-based Ethanol and U.S. Energy Independence
Coach: Jessy Friedman
Team: **Michael Bacsik, Jephthah Liddie, Joshua Newman, Thomas Sozzi, Kevin Tien**
Fifth Place, Exemplary Team Prize of $5,000:
Hunterdon Central Regional High School, Flemington, NJ – Team #141

Ethanol: The Future of America's Energy Independence?
Coach: David Gelb
Team: **Brandon Comella, Gawain Lau, Kelvin Mei, Nevin Raj, Yiwen Zhan**

The top six winning solutions, photo galleries, and webcasts of the presentations and awards ceremony are available on the M3 Challenge website. To view these, and to find more information on the Challenge, visit http://m3challenge.siam.org/.

Congratulations to all the students who participated. Keep up the good work!

Greetings from the Student Delegate President!

I'd like to welcome you to a new year as your Student Delegate President, hoping for a very exciting season of Mu Alpha Theta.

Working with me this year are the Vice-President **Alexa Tavasci** (Thomas Jefferson High School, WA), Secretary/Treasurer **Linda Pan** (Lakota West High School, OH), and Parliamentarian **Trang Le** (Alfred M. Barbe High School, LA). They have been very helpful so far, and I'm sure will continue to be throughout the year.

After a quick break following convention, we got right to work forming committees and setting goals. The Communications Committee will be continued this year, chaired by Linda Pan and myself. We have started creating the student delegate newsletter to be distributed later this year. Also, we have completely redone the website, which has a new home at www.maostudents.org. The Recruitment Committee, chaired by Trang Le, has also begun work on encouraging more attendance to the National Convention from existing chapters. The "Fun" Committee, headed by Alexa Tavasci is aimed at creating events at next year's convention like mixers and dances, which will be amazingly fun. If you would like to join one of these committees or would like to start a new one, email any of the committee heads.

Thank you all for your involvement in Mu Alpha Theta! I am very excited to serve as your student delegate president for this year and look forward to a great convention next year. See you in Knoxville!
Logan Stafman
Student Delegate President
Lawton Chiles High School

THE ACTUARIAL FOUNDATION's PROJECT MATH MINDS SCHOLARSHIP CONTEST

The Actuarial Foundation is pleased to announce it will run its Project Math Minds contest for another year. This unique scholarship opportunity highlights the high level math skills used by the actuarial profession. **Students compete for college scholarships between $1000 - $5000 by successfully completing an actuarial research project.**

For more information visit: http://www. actuarialfoundation.org/ youth/math_minds.html.

Last year, **Bryan Taylor McMeen** of Virgil I. Grissom High School in Huntsville, Alabama received a $5000 Project Math Minds scholarship for his project, *A Statistical Analysis Regarding the Projected Growth of Worker's Compensation Insurance*.

Jeremy Fogg, the project chair, was pleased with the results. "The actuarial field is the science and art of analyzing information that have complex relationships to assist financial planning for the near future. We were pleasantly surprised with the quality of work submitted by Mr. McMeen. He is proof that our youth are capable of using technology to analyze a large amount of data, providing a reasonable recommendation and solid support justifying his recommendations."

'Actuaries' are consistently rated as one of the top math related jobs in America. Check out these websites for more about this exciting field: http://www.bls.gov/oes/current/oes152011.htm and http://www.BeAnActuary.org.

Summer at Stanford

I participated in Stanford's Education Program for Gifted Youth this summer, and it was the best 3 weeks of my life! I studied Topology and Knot Theory, which was very interesting. I learned about everything from different kinds of knots to how to assign polynomials to the knots. It was a very informative class, and I really enjoyed the challenging material. I feel that I learned an incredible amount of material for such a short period of time. For our final project, we learned several proofs related to coloring, which was one of the highlights of the program.

We had class for two-and-a-half hours each day and study hall for three hours. Other than that, we did a lot of activities on Stanford's beautiful campus, such as hiking, sports, tie-dying, and fountain hopping (a Stanford tradition). It was such a great experience to meet other students from across the U.S. and across the world who were all as smart as, if not smarter than, me. I thoroughly enjoyed my time at Stanford this summer, and I hope to attend it for college in a little over a year. Thank you for making it possible for me to attend this life-changing program.

Thank you again,
Natalie Cooper

(Natalie was a recipient of a 2008 Mu Alpha Theta Summer Grant. For more info on Summer Grants, visit our website at www.mualphatheta.org > Scholarships > Summer Grants.)

New Tutoring Opportunity!

Mu Alpha Theta is excited to offer its members, in partnership with the National Science and Technology Education Partnership, NSTEP, an outreach opportunity to commence in fall 2008. **Study Buddy** is a cutting edge web based online peer-to-peer tutoring program designed to address low math scores of underperforming middle and high school students. This homework assistance program utilizes the latest interactive, collaborative technology to bring students from different locations across the country together for online tutoring sessions.

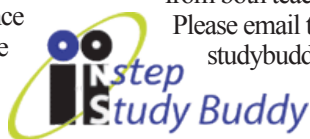**Students** -As a Mu Alpha Theta member, you will be paired with students who are struggling and may not otherwise have access to a tutor due to economic and/or geographic reasons. Your participation will be tracked by software and will serve as community service credit hours.

Teachers - We encourage you to enroll your Mu Alpha Theta members in Study Buddy and can do so with minimal effort! Detailed reports of tutor activity are available to enable you to monitor and log your students' participation in the program.

NSTEP welcomes inquiries from both teachers and students. Please email them directly at studybuddy@nstep-online.org with questions about how you can integrate Study Buddy into your classroom or Mu Alpha Theta chapter! For additional information, please visit our home page at http://www.nstep-online.org/studybuddy/default.asp. Tutor registration will begin September 18, 2008.

Lakota West Student Recognized As Presidential Scholar

Lakota West High Senior **Raymond Xi** of West Chester Township, OH was awarded the national recognition of Presidential Scholar. He joined three other students in Ohio along with 139 others across the country for this year's designation.

Raymond participated at Lakota West as the Captain of the Chess Team, Captain of the Junior Engineering Technology Society Team, Mu Alpha Theta President and Captain for the Academic Quiz Team. He also served as the Co-Captain of the Science Olympiad and Group Leader for Peer Counseling as well as played on the school's Ultimate Frisbee Team. He has been dedicated to the community through the Junior Classic League as well as fundraising for the American Cancer Society. This fall Raymond is attendingYale to study mathematics and economics.

The United States Presidential Scholars Program was established in 1964 to recognize and honor some of our Nation's most distinguished graduating high school seniors. Each year, up to 141 students are named as Presidential Scholars, one of the Nation's highest honors for high school students.

Students have the opportunity to become Presidential Scholars based on scholastic ability and broad academic achievement or on the basis of their academic and artistic scholarship in the visual arts, the performing arts, or creative writing.

Congratulations to Raymond and all of this year's Presidential Scholars!

A TEN DAY SYLLABUS FOR PRECALCULUS

DAY 1: Teach them that $(a+b)/c$ is $(a/c) + (b/c)$
DAY 2: Teach them that $a/(b+c)$ is NOT $(a/b) + (a/c)$
DAY 3: Teach them that $x / \ln(x)$ is NOT "$1 / \ln$"
DAY 4: Teach them that you can't solve $(\sin(kx)) = 1$ by saying "$x = 1/\sin(k)$"
DAY 5: Remind them that $a/(b+c)$ is NOT $(a/b) + (a/c)$
DAY 6: Show them a movie of a student sitting in a field, writing "$(a+b)^2 = a^2 + b^2$" and then getting HIT BY A TRAIN
DAY 7: Remind them that $a/(b+c)$ is NOT $(a/b) + (a/c)$
DAY 8: Teach them that if the domain of the a function f is the reals, the graph of $y = f(x)$ is NOT a blank pair of axes and perhaps they should adjust the "window"
DAY 9: Teach them that $x/(y+z)$ is NOT $(x/y) + (x/z)$
DAY 10: Group work: Bring a trout to class. Have them solve $\sin(kx) = 1$.
If they get $x = 1/\sin(k)$, hit them with the trout. Make it a big trout.

A Word from the Student Delegate Vice President

Hey everyone! Hope you all are doing well and that you enjoyed the last part of your summer after convention (and aren't getting as much rain as Seattle.) This year we spent five days together in sunny Sacramento, not only competing but creating new friendships.

Every morning we woke up and nervously waited for the doors to open, to take the test that might get us up on the stage with a trophy in hand. But, before and after, we filled our time networking. Wherever we were, at the pool playing mafia or during lunch and dinner, we found people who shared the same interests and even were going to the same college.

Early on, the convention showed it was never going to be boring. From the first night of excitement at the carnival, where all the guys were throwing their friends upside down on the Velcro climbing wall while the girls frantically asked the fortune tellers about their future husbands, we knew how much fun it was going to be. It started the competition off with us laughing.

Now we all live with the memories as we go into the next school year preparing for more competitions to come. Your officer team is excited and ready to make this a great year and hope to help make next year's convention even more memorable. Keep in touch with those you met and if you have any input don't be shy! Contact any of us at www.maostudents.org. Enjoy your year!

Alexa Tavasci
Student Delegate Vice President

POINT-SET TOPOLOGY AS A TOOL TO BUILD CRITICAL THINKING SKILLS

A topology is a set that has certain properties. Point-set topology (or general topology) is a branch of mathematics that deals with the study of sets and topologies. The concepts can be useful in a wide range of contexts.

Topology is often not introduced in a standard mathematics program until late undergraduate study. There are some good reasons for this. However, there are benefits from earlier exposure to topology, particularly in the context of standardized examination systems as they exist today.

In particular point-set topology is an excellent tool that teaches analytical problem solving skills. It tends to develop an appreciation for complete and thorough understanding of topics in a way that standardized exams in topics like Algebra and Calculus sometimes do not. It is an especially useful tool in teaching critical thinking skills that apply to arbitrary problems in a wide range of contexts.

When using point-set topology as a tool in this setting, the fact that the language is in many cases unfamiliar to the beginning student is a benefit that can be leveraged – rather than a deficiency.

What this article aims to do is present a self-contained introduction to point-set topology directed towards a primary audience of early undergraduate and advanced high school mathematics students with no background in the subject.

Definition A **set** is a collection of things. The things in it are called **elements**.

Example {1, 2, 3} is a set with three elements. Its elements all happen to be numbers: 1, 2, and 3. { {1}, 2, {1, 2} } is a set which also has three elements. One of its elements is {1}, a set (which has one element, the number 1). One of its elements is a number, 2. One of its elements is a set {1, 2}, which has two elements. Sets can be elements of sets.

Definition When a set has no elements it is called the **empty set**, denoted $\varnothing$. It is useful in a way similar to how having the number 0 can be useful in arithmetic.

Definition A set Y is a **subset** of X when all the elements in Y are elements of X. This is also sometimes worded Y is **contained** in X. This is written $Y \subset X$.

Example $\{1, 2, 3\} \subset \{1, 3, 4, 2\}$. Each element of the LHS (left hand side) is an element of the RHS.

Example $\{1, 2, 3\} \not\subset \{1, 2, \{3\}\}$. $3 \in$ LHS, but $3 \notin$ RHS.

Definition For real numbers a, b with a < b define $(a, b) = \{x \mid a < x < b\}$ and define $[a, b] = \{x \mid a \leq x \leq b\}$

Example Show $(3, 6) \subset [2, 7]$

Let x ∈ LHS. Then $3 < x < 6$. Since $2 \leq 3$ and $3 < x$, we have $2 \leq x$.
Since $x < 6$ and $6 \leq 7$, we have $x \leq 7$. So $2 \leq x \leq 7$. So x ∈ RHS.

We have shown that any element of the LHS must be an element of the RHS, so we have shown $(3, 6) \subset [2, 7]$.

Definition The **intersection** of sets A and B is $A \cap B = \{x \mid x \in A \text{ \textbf{and} } x \in B\}$
This is read as "the set of all x such that x is an element of A **and** x is an element of B"]
The **union** of sets A and B is $A \cup B = \{x \mid x \in A \text{ \textbf{or} } x \in B\}$
This is read as "the set of all x such that x is an element of A **or** x is an element of B"]

Example $\{1, 2, 3\} \cap \{3, 4\} = \{3\}$

Example $\{1, 2, 3\} \cup \{3, 4\} = \{1, 2, 3, 4\}$

Example Let X be a set. Show $X \cap \varnothing$ is the empty set.
$X \cap \varnothing = \{x \mid x \in X \text{ and } x \in \varnothing\}$. There is no x ∈ ∅.
So there is no x such that x ∈ X and x ∈ ∅. So $X \cap \varnothing$ has no elements. Therefore, $X \cap \varnothing = \varnothing$.

Definition Two sets A and B are said to be equal $(A = B)$ when $A \subset B$ and $B \subset A$.

Exercise Let X be a set. Show $X \cup \varnothing = X$.
Hint: Show that any element of the LHS is an element of the RHS.
Then show that any element of the RHS is an element of the LHS.

Example Let X be a set. Show $\varnothing \subset X$.
$\varnothing \subset X$ means that all elements of ∅ are elements of X. Suppose ∅ is not a subset of X. That is, suppose that not all elements of ∅ are elements of X. That would mean that there is some element of ∅ that is not an element of X. But that would mean that there is some element of ∅. But ∅ is the set with no elements, so our assumption has led to a contradiction. So our assumption that ∅ is not a subset of X is false. So $\varnothing \subset X$.

Exercise Let X be a set. Show $X \subset X$.

Exercise Let $A, B \subset X$. Show $A \cap B \subset X$.

Example Suppose we have a bunch of sets, each contained in X.
Show that the union of all these sets is also contained in X.

Choose x in the union of these sets. Then x is in at least one of these sets, by the definition of the union. Choose one of these sets that x is in, and call it A. Note A is contained in X. So we have $x \in A \subset X$. So x ∈ X. We have shown that any x in the union of these sets must be in X. So we have shown that the union is contained in X.

Definition When $Y \subset X$, we define the **complement** of Y in X as
$X - Y = \{x \in X \mid x \notin Y\}$

Example Let $Y = \{1, 2, 3\}$, $X = \{1, 3, 4, 2\}$. Then $X - Y = \{4\}$.

Exercise Show $X - \varnothing = X$.

Definition A **topology** on a set X is a set T whose elements are subsets of X having three properties outlined below. The elements of the topology (which are all sets) are called **open** sets.
    1) $\varnothing$ (the empty set) and X are open.
    2) The intersection of any two open sets is open.
    3) The union of any collection of open sets is open.

A set X equipped with a topology T is sometimes referred to as a **topological space**.

Example Let X be any set. Define $T = \{A \mid A \subset X\}$. That is, T is the set of all sets A that are contained in X. T is a topology on X. Why?

We already showed that $\varnothing \subset X$. So $\varnothing$ is open in T.
We already showed that $X \subset X$. So X is open in T.

Suppose $A \subset X$ and $B \subset X$. We already showed that $A \cap B \subset X$. So $A \cap B$ is open.

Suppose we have a bunch of open sets, each contained in X. We showed above their union is also contained in X. So their union is open.

Comment We now have a way of defining a topology on any arbitrary set. The set could contain numbers, or sets, or cars. It could contain anything. When we define a topology T on X so that $T = \{A \mid A \subset X\}$, we say X is equipped with the **discrete** topology.

Definition Let X be a set equipped with some topology T and let $A \subset X$. We say A is **closed** in X when $X - A$ is open.

Exercise Let $X = \{1, 2, 3, 4\}$. Equip X with the discrete topology.
What is $X - \{2, 3\}$? Explain why the set $\{2, 3\}$ is open in X.
Explain why $X - \{2, 3\}$ is open in X. Explain why $\{2, 3\}$ is closed in X.

Exercise Let $X = \{1, 2, 3\}$ be equipped with the discrete topology. Which subsets of X are open and closed? Open but not closed? Closed but not open?

Exercise Let $X = \varnothing$ be equipped with the discrete topology. Which subsets of X are open and closed? Open but not closed? Closed but not open?

Exercise Let X = {a} (i.e. X has a single element) be equipped with the discrete topology. Which subsets of X are open and closed? Open but not closed? Closed but not open?

Exercise Let X be any set with more than one element and equip X with the discrete topology. Which subsets of X are open and closed? Open but not closed? Closed but not open?

Exercise Let X be any set. Define T = {∅, X}. Note that the only open sets are ∅ and X. Show that T is a topology on X.

Definition When a set X is equipped with the topology above, T = {∅, X}, we say X is equipped with the **indiscrete** topology.

Exercise Let X be a set equipped with the indiscrete topology. Which subsets of X are open and closed? Open but not closed? Closed but not open?

Exercise Let X = {0, 1} and let T = {∅, X, {0}}. Show that T is a topology.

Definition The set X = {0, 1} with topology T = {∅, X,{0}} is the **Sierpinski space**.

Exercise In the Sierpinski space, which sets are open and closed? Which sets are open but not closed? Which sets are closed but not open?

Comment Consider the real numbers. Recall this includes 3, -2, 0, 1/5, π, -2.6783, etc. Recall (a, b) = {x | a < x < b}. This is sometimes called the **open interval** (a, b).

Exercise Consider the real numbers. Define T = {X is an open interval (a, b)}. Show that T is not a topology on the real numbers. Hint: Examine (1, 2) $\cup$ (3, 4).

Summary
Point-set topology is about defining topologies on sets and studying properties associated with them. Its language often leads towards providing formal language to describe the notions of "similarity" and "closeness". Its study can be used as a tool to build critical thinking skills applicable in a wide range of contexts.

For further study, one standard text these days is: Topology, by James Munkres. A student with zero background may find the following more approachable: Topology and the Language of Mathematics by Chris Cunliffe; Basic Topology by M.A. Armstrong; or the Schaum's Outline of General Topology, by Seymour Lipschutz.

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