#1 Theta Ciphering
MAΘ National Convention 2021

$$M = \left(\sqrt{13} + \sqrt{14} + \sqrt{15}\right)\left(\sqrt{13} + \sqrt{14} - \sqrt{15}\right)$$

$$U = \left(\sqrt{13} - \sqrt{14} + \sqrt{15}\right)\left(-\sqrt{13} + \sqrt{14} + \sqrt{15}\right)$$

What is the value of **MU**?

Wiggie chooses 3 integers without replacement from between 1 and 10 inclusive. The probability they can be arranged to form an arithmetic sequence is $\frac{J}{120}$. What is J?

In a geometric series of real numbers, the sum of the first two terms is 7 and the sum of the first six terms is 91. If r is the common ratio, what is the value of r^2 ?

$_nC_1, {}_nC_2$, and $_nC_3$ forms an arithmetic sequence with $n > 3$. What is the value of n?

Through a point on the hypotenuse of a right triangle, Lines are drawn parallel to the legs of the triangle so that the triangle is divided into a square and two smaller right triangles. The area of one of the two small right triangles is 3 times the area of the square. What is the ratio of the area of the square to the area of the other small right triangle?

Trapezoid $WXYZ$ has $\overline{WZ}$ parallel to $\overline{XY}$,
$XZ = 3$, $m\angle ZXW = 27°$, and $m\angle XZY = 54°$.
The ratio of $XY:WZ$ is $7:4$. What is YZ?

Given: $5 = 17k^{\frac{-1}{4}} - 6k^{\frac{-1}{2}}$. What is the product of the solutions?

Compute $\left(\dfrac{-1}{2} + \dfrac{\sqrt{3}}{2} i \right)^{2021}$.

Compute $\begin{vmatrix} \log_{128} \dfrac{1}{5} & \log_{1024} \dfrac{1}{49} \\ \log_{49} 32 & \log_{125} 64 \end{vmatrix}$.

Each of the letters M, R. L, and U represent a different odd integer between 2 and 10. What is the least possible value of $\dfrac{M \bullet R - L}{U}$?

What is the area of the quadrilateral formed by connecting the 4 intersection points of the curves $x^2 + 4y^2 = 6x + 8y + 67$ and $x^2 = 6x + 8y + 31$?

Mr. Lu has a gallon of David's sweet tea. He drinks 1/3 of it and then fills back to the top with water and stirs and then drinks 1/3 of the contents. He repeats this process until he has consumed one gallon of liquid. What part of the original gallon of tea remains?