0. π
1. $\frac{-1}{2}$
2. $\frac{5}{4}$
3. 12
4. $\sqrt{17}$
5. 204
6. 12
7. 1011
8. $\frac{-8}{5}$
9. 100
10. $\frac{1}{9}$
11. $\frac{2\sqrt{3}}{3}$
12. 4

Alpha ciphering solutions Nationals 2021

0.

$$\left(\cos^2 x+\sin^2 x\right)\left(\cos^2 x-\sin^2 x\right)=\left(\cos^2 x-\sin^2 x\right)=\cos 2x\rightarrow\frac{2\pi}{2}=\pi$$

$$k\left(x^2-x-k-1\right)=x\left(kx-x-k+1\right)$$

$$kx^2-kx-k^2-k=kx^2-x^2-kx+x$$

1. $x^2-x-k^2-k=0\rightarrow D=0=b^2-4ac=1-4\left(-k^2-k\right)$

$$4k^2+4k+1=0\rightarrow\left(2k+1\right)^2=0\rightarrow k=\frac{-1}{2}$$

$$r-d+r+r+d=3r=\frac{144}{64}\rightarrow r=\frac{3}{4}$$

2. $\frac{3}{4}\left(\frac{9}{16}-d^2\right)=\frac{15}{64}\rightarrow\frac{9}{16}-d^2=\frac{5}{16}\rightarrow d=\pm\frac{1}{2}\rightarrow\frac{3}{4}+\frac{1}{2}=\frac{5}{4}$

$$\frac{d}{N}+\left(N-2\right)\left(\frac{d}{N}+8\right)=d+72$$

3. $\frac{d}{N}+d+8N-\frac{2d}{N}-16=d+72$

$$8N-\frac{d}{N}=88\rightarrow 8N^2-88N=d\rightarrow 8N\left(N-11\right)=d\rightarrow N=12$$

$$\left(2x\right)^2+y^2=9$$

4. $x^2+\left(2y\right)^2=\frac{49}{4}\rightarrow 5x^2+5y^2=\frac{85}{4}\rightarrow x^2+y^2=\frac{17}{4}$

$$\left(ZL\right)^2=4x^2+4y^2=17\rightarrow ZL=\sqrt{17}$$

5. ${}_{10}C_3+{}_9C_3=\frac{10\bullet 9\bullet 8+9\bullet 8\bullet 7}{3\bullet 2\bullet 1}=120+84=204$

6. Using product of chord segments you get $6 \bullet 24 = 12 \bullet 12$. So the longest chord is the diameter at 30 and the shortest chord would be24.There is only 1 shortest chord and 1 longest chord but chords of length 25,26,27,28, and 29 can occur twice. That is a total of 12

7.
$$\sin 90 + \cos 90 = 1$$
$$\sin^2 180 + \cos^2 180 = 1$$
$$\sin^3 270 + \cos^3 270 = -1$$
$$\sin^4 360 + \cos^4 360 = 1$$

You can see that this will recycle in blocks of 4 with each block summing to 2. So we have 505 blocks of 4 summing to 1010 plus an extra 1 for a total of 1011

8. $\left(\tan\frac{1}{5}x - \tan x\right)\left(\cos\frac{1}{5}x\cos x\right) = \sin\frac{k}{2}x \to \sin\frac{1}{5}x\cos x - \sin x\cos\frac{1}{5}x$

$$\sin\left(\frac{1}{5}x - x\right) = \sin\frac{k}{2}x \to \frac{-4}{5} = \frac{k}{2} \to k = \frac{-8}{5}$$

9.
$$2^{2023} - 2 = \left(2^0 + 2^1 + 2^2 + \ldots + 2^{2021}\right)\log x$$
$$2\left(2^{2022} - 1\right) = \left(2^{2022} - 1\right)\log x \to \log x = 2 \to x = 100$$

10.
$$y + 4 - \frac{3}{2} = \frac{-3}{2}\left(x^2 - 2x + 1\right) \to \frac{-2}{3}\left(y + \frac{5}{2}\right) = (x-1)^2$$
$$A = \frac{1}{2}(4p)(2p) = 4p^2 \to 4p = \frac{2}{3} \to p = \frac{1}{6} \to 4p^2 = \frac{1}{9}$$

11.
$$\cos x = \frac{Y^2 + Z^2 - X^2}{2YZ} \to Y^2 + Z^2 + 2YZ = 4X^2$$
$$\cos x = \frac{X^2}{2X^2} = \frac{1}{2} \to \csc x = \frac{2\sqrt{3}}{3}$$

12. Since left hand side is real the right-hand side is real which makes a=0.

$$\sqrt{b^2+3^2}=1+b\rightarrow b^2+9=b^2+2b+1\rightarrow 2b=8\rightarrow b=4\rightarrow b-a=4-0=4$$