#1 Alpha School Bowl
MAΘ National Convention 2021

$$A = \sum_{n=10}^{4052} 2021$$

$$B = \sum_{n=1}^{2021} n$$

$$C = \sum_{n=1}^{2021} n^2$$

$$D = \sum_{n=1}^{2021} n^3$$

Find $\frac{C}{A} + B^2 - D$

Let $\vec{v} =< 1, 2, 2 >$ and let $\vec{w} =< 2, 3, 6 >$

Let X be the cosine of the acute angle between these vectors.

Let Y be the magnitude of the cross product between these vectors.

Let $Bx + Cy - z + D = 0$ be the equation of the plane containing these vectors and the point $(20,2,1)$

Let $\frac{x-E}{F} = y = \frac{z-G}{H}$ be the symmetric equations of the line in the direction of $\vec{w}$ containing the point $(2,0,21)$.

Find $7X + 2Y^2 + B + C + D + E + F + G + H$

Anagh wishes to form a committee with seven people. If he has three Thetas, five Alphas, and four Mus to choose uniformly at random from, let A be the probability that the committee has two Thetas, three Alphas, and two Mus.

The probability of Luke knowing how to solve Question #30 on a FAMAT test is $\frac{9}{10}$. If he doesn't know how to solve the question, he will guess one of the five choices with equal likelihood. If he does know how to solve the question, he will always get it correct. Given that Luke gets Question #30 correct, let B be the probability that Luke guessed.

Assume in a certain population that among all twin births, 60% of them are of the same gender. Assume that identical twins will always be the same gender, and that non-identical twins will be the same gender only 50% of the time. Let C be the proportion of twin births that are identical twins in this population.

Find $\frac{1}{AC} + \frac{1}{B}$.

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Let $A = \lim_{x \to 1} \frac{x^2 - 2x + 1}{x^2 - 1}$

Let $B = \lim_{x \to \infty} \frac{x^2 - 2x + 1}{x^2 - 1}$

Let $C = \lim_{x \to 1} \frac{x^3 + x^2 - 2}{3x^3 - 3}$

Let $D = \lim_{x \to \infty} \frac{x^3 + x^2 - 2}{3x^3 - 3}$

Find $A + B + C + D$

Alan and Srijan are running around a 21m circular track. Alan runs clockwise at 43 m/hr and Srijan runs counterclockwise at 47 m/hr. They start at the same spot on the track and run for 5 hours. Let A be the number of times they pass each other after they start running.

John only owns two types of books: comic books and math books. 1/6 of his books are comic books. After going to a book sale, he buys 10 more comic books, so 1/4 of his books are now comic books. Let B be the total number of books he has after the book sale.

Henry is trying to decide whether to take the stairs or the elevator. If he takes the stairs, it will take him 25 seconds to walk up each flight of stairs. If he takes the elevator, he will have to wait for 4 minutes for the elevator to arrive, after which it will take 5 seconds to move up each floor. Let C be the minimum number of floors for which taking the elevator takes less time than taking the stairs.

Find $A + B + C$

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Let A be the maximum possible value of the determinant of $\begin{bmatrix} 2x & 1-x & 0 \\ 2-x & 1 & 1 \\ 2 & 0 & 2 \end{bmatrix}$.

Consider the system of equations:

$$\begin{aligned} x + 4y - 2z + 3w &= 2 \\ 2x - y - z + 15w &= 10 \\ 3x + y - 5z + 6w &= 4 \\ 4x + 2y + z + 9w &= 6 \end{aligned}$$

Let B be the value of w in the solution to this system.

Let C and D be the eigenvalues of $\begin{bmatrix} 2 & 9 \\ 4 & 2 \end{bmatrix}$.

Find $AB + C + D$

Let A be the number of distinct arrangements of the word *COMBINATORICS*

Let B be the number of positive integer solutions of the form (X_1, X_2, X_3) to the equation

$$X_1 + X_2 + X_3 = 2021$$

Let C be the number of terms in the expansion of $(X_1 + X_2 + X_3)^{2021}$

Dr. Santos has 16 (indistinguishable) black Math Competition shirts and 5 (indistinguishable) gold Math Competition shirts. Let D be the number of ways he can hang these shirts in his (linear) closet so that each gold shirt is separated by at least two black shirts. Gold shirts may be at the leftmost or rightmost end.

Find:

$$\frac{1}{5!}\frac{A}{D} + \frac{{}_{2023}P_5}{BC}$$

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Let $A = \sum_{n=0}^{\infty}\left(\frac{1}{2021^n}\right)$ Let $B = \sum_{n=1}^{\infty}\left(\frac{n}{2021^n}\right)$

Let $C = \sum_{n=2021}^{\infty}\left(\frac{1}{n^2+n}\right)$ Let $D = \sum_{n=2}^{2020}\left(\ln\left(\frac{n-1}{n+1}\right)\right)$

Find:

$$\frac{2Be^{-D}}{A} + \frac{1}{C}$$

Let $A = \sin(2\alpha)$ if

$$\frac{\pi}{4} = \sin(\alpha + \sin(\alpha + \sin(\alpha + \cdots)))$$

Let B be the sum of all values of $0 < \theta < \pi$ for which

$$\frac{1+\sqrt{5}}{2} = \frac{\tan(\theta)}{1 - \frac{\tan(\theta)}{1 - \frac{\tan(\theta)}{1 - \cdots}}}$$

Let $C = \ln\left(e^{\frac{\pi i}{2} + 2\ln\left(e^{\frac{\pi i}{2} + 2\ln\left(e^{\frac{\pi i}{2} + 2\ln(\cdots)}\right)}\right)}\right)$ if the arguments of complex numbers are limited to $(-\pi, \pi]$.

Find $A + B^2 + C^2 + 1$

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Let A be the area contained within the graph of $\frac{(x-2021)^2}{25} + \frac{(y+2021)^2}{9} = 1$

Let B be the area contained within the graph of $|x| + |y| = 21$

Let C be the area contained within the polar graph of $r = 20\sin(\theta)$

Let D be the area of the rectangle with points on the ellipse $\frac{x^2}{169} + \frac{y^2}{25} = 1$ that has as two of its sides the latera recta of this ellipse.

Find $A + B + C + 13D$

Consider the conic section $rx^2 + (1-r)xy + y^2 + 2y - 2 = 0$ for real r.

Let A be the sum of all values of r for which this conic is a circle.

Let B be the sum of all values of r for which this conic is a parabola.

Let C be the sum of all values of r for which this conic is degenerate.

Using rotation of variables, this conic can be written in the form $A'x'^2 + C'y'^2 + D'x' + E'y' + F' = 0$ in the rotated x'-y' plane. Let θ be any angle of rotation for which this will be true for all values of $r \neq 1$, and let $D = \sin^2(\theta)\cos^2(\theta)$.

Find $A + B + CD$

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Let A be the sum of the real solutions to:

$$2\log_4(x) + \log_4(x^2 - 2x + 1) = 1$$

Let B be the number of distinct real solutions to:

$$(x^3 + 2x^2 - x - 1)^{(x^2+5x+4)} = 1$$

Let C be the sum of the real solutions for $0 < x \leq 2\pi$ to:

$$2\cos^3(x) - 5\cos^2(x) + \cos(x) + 2 = 0$$

Find $A + B + C$

Let X be the binary representation of a number such that

$$21_{20} + 20_{21} + 20_{2021} + 2021_3 = X_2$$

How many times does the digit 1 appear in X?

Evaluate:

$$\frac{2\sqrt{\sec^2\left(\frac{\pi}{24}\right)-1}}{1-\frac{1-\cos\left(\frac{\pi}{12}\right)}{1+\cos\left(\frac{\pi}{12}\right)}}$$