Mu School Bowl
Test #331
Question #0

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Test #331
Question #0

#0 Mu School Bowl
MAΘ National Convention 2021

Let $A =$ the number of non-distinct permutations of CAPTAHAB.

Let $B =$ the number of distinct permutations of CAPTAHAB.

Let $C =$ the number of platonic solids.

Let $D =$ the eccentricity of $x^2 - 3y + 7x - 8 = 0$.

Find the value of $\frac{A}{B} + C + D$.

#0 Mu School Bowl
MAΘ National Convention 2021

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Find the value of $\frac{A}{B} + C + D$.

Mu School Bowl
Test #331
Question #1

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Test #331
Question #1

#1 Mu School Bowl
MAΘ National Convention 2021

Let $f(x) = 2021 - 12x^3 - 6x$. Let R be the finite region bounded by $f(x)$, the x-axis, $x = 0$, and $x = 2$.

Let A be the value obtained when the area of R is approximated using a Left-handed Riemann Sum with 8 equal subintervals.

Let B be the value obtained when the area of R is approximated using a Right-handed Riemann Sum with 8 equal subintervals.

Let C be the value obtained when the area of R is approximated using the Trapezoidal Rule with 8 equal subintervals.

Let D be the value obtained when the area of R is approximated using Simpson's Rule with 8 equal subintervals.

Find $A + B - 2C + D$

#1 Mu School Bowl
MAΘ National Convention 2021

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Find $A + B - 2C + D$

Mu School Bowl
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Question #2

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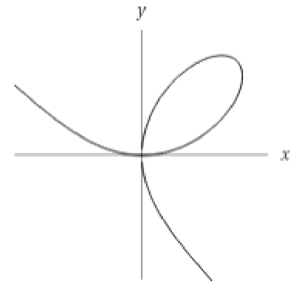
Question #2

#2 Mu School Bowl
MAΘ National Convention 2021

Consider the Folium of Descartes pictured at right with the equation

$$x^3 + y^3 = 3axy; \quad a > 0$$

Let (A, B) be the coordinates of the point in the first quadrant that is furthest from the origin.



Let (C, D) be the coordinates of the point with a horizontal tangent line that has the largest y-value.

Let (E, F) be the coordinates of the point with a vertical tangent line that has the largest y-value.

Let $y = Hx + G$ be the equation of the oblique asymptote of this curve.

Find

$$\frac{A}{a} + \frac{B}{a} + \left(\frac{C}{a}\right)^3 + \left(\frac{D}{a}\right)^3 + \left(\frac{E}{a}\right)^3 + \left(\frac{F}{a}\right)^3 + \frac{G}{a} + H$$

#2 Mu School Bowl
MAΘ National Convention 2021

Consider the Folium of Descartes pictured at right with the equation

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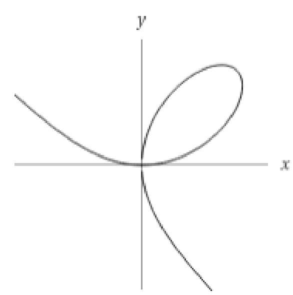
Let (A, B) be the coordinates of the point in the first quadrant that is furthest from the origin.



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Let (E, F) be the coordinates of the point with a vertical tangent line that has the largest y-value.

Let $y = Hx + G$ be the equation of the oblique asymptote of this curve.

Find

$$\frac{A}{a} + \frac{B}{a} + \left(\frac{C}{a}\right)^3 + \left(\frac{D}{a}\right)^3 + \left(\frac{E}{a}\right)^3 + \left(\frac{F}{a}\right)^3 + \frac{G}{a} + H$$

Mu School Bowl
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Question #3

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Question #3

#3 Mu School Bowl
MAΘ National Convention 2021

Alex wishes to form a committee with seven people. If he has three Thetas, five Alphas, and four Mus to choose uniformly at random from, let A be the probability that the committee has two Thetas, three Alphas, and two Mus.

The probability of Saathvik knowing how to solve Question #30 on a FAMAT test is $\frac{9}{10}$. If he doesn't know how to solve the question, he will guess one of the five choices with equal likelihood. If he does know how to solve the question, he will always get it correct. Given that Saathvik gets Question #30 correct, let B be the probability that Saathvik guessed.

Assume in a certain population that among all twin births, 60% of them are of the same gender. Assume that identical twins will always be the same gender, and that non-identical twins will be the same gender only 50% of the time. Let C be the proportion of twin births that are identical twins in this population.

Find $\frac{1}{AC}+\frac{1}{B}$.

#3 Mu School Bowl
MAΘ National Convention 2021

Alex wishes to form a committee with seven people. If he has three Thetas, five Alphas, and four Mus to choose uniformly at random from, let A be the probability that the committee has two Thetas, three Alphas, and two Mus.

The probability of Saathvik knowing how to solve Question #30 on a FAMAT test is $\frac{9}{10}$. If he doesn't know how to solve the question, he will guess one of the five choices with equal likelihood. If he does know how to solve the question, he will always get it correct. Given that Saathvik gets Question #30 correct, let B be the probability that Saathvik guessed.

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Find $\frac{1}{AC}+\frac{1}{B}$.

Mu School Bowl
Test #331
Question #4

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Test #331
Question #4

#4 Mu School Bowl
MAΘ National Convention 2021

Let $A = \frac{d}{dx}\left[\sqrt{25-x^2}\right]\Big|_{x=3}$

Let $B = \frac{d}{dx}\left[(x+1)(x^2+1)(x^3+1)\right]\Big|_{x=1}$

Let $C = \frac{d}{dx}\left[\ln(\sec(x)+\tan(x))\right]\Big|_{x=\frac{\pi}{4}}$

Let $D = \frac{d}{dx}\left[\arctan(cos(x))\right]\Big|_{x=\frac{\pi}{4}}$

Find $A \cdot B \cdot C \cdot D$

#4 Mu School Bowl
MAΘ National Convention 2021

Let $A = \frac{d}{dx}\left[\sqrt{25-x^2}\right]\Big|_{x=3}$

Let $B = \frac{d}{dx}\left[(x+1)(x^2+1)(x^3+1)\right]\Big|_{x=1}$

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Let $D = \frac{d}{dx}\left[\arctan(cos(x))\right]\Big|_{x=\frac{\pi}{4}}$

Find $A \cdot B \cdot C \cdot D$

Mu School Bowl
Test #331
Question #5

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Question #5

#5 Mu School Bowl
MAΘ National Convention 2021

Let $A = \frac{3}{2}\int_0^{2020} (x-1010)^5 + (x-1010)^3 + (x-1010)^2\, dx$

Let $B = \int_0^{2021} \frac{x^{2021}}{x^{2021}+(2021-x)^{2021}}\, dx$

Let $C = \int_0^{\pi} \frac{x\sin(x)}{1+\cos^2(x)}\, dx$

Let $D = \int_0^{\infty} \frac{2\ln(x)}{x^2+4}\, dx$

Find

$$\sqrt[3]{A} + 2B + \frac{D}{\sqrt{C}}$$

#5 Mu School Bowl
MAΘ National Convention 2021

Let $A = \frac{3}{2}\int_0^{2020} (x-1010)^5 + (x-1010)^3 + (x-1010)^2\, dx$

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Let $D = \int_0^{\infty} \frac{2\ln(x)}{x^2+4}\, dx$

Find

$$\sqrt[3]{A} + 2B + \frac{D}{\sqrt{C}}$$

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Test #331
Question #6

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Question #6

#6 Mu School Bowl
MAΘ National Convention 2021

$f(x)$ and $g(x)$ are continuous, twice-differentiable functions. Using only the information in the table below, find the indicated values.

$x =$	**1**	**2**	**3**	**4**
$f(x)$	2	3	4	9
$f'(x)$	3	1	5	12
$g(x)$	-1	4	3	10
$g'(x)$	5	5	-1	2

Let $A = h'(1)$ if $h(x) = \frac{g(x)}{f(x)+1}$

Let $B = k'(2)$ if $k(x) = \sqrt{f\left(g(x)\right)}$

Let C be the average rate of change of $p(x) = f(x)g(x)$ on the interval [1,3]

Let D be the value of c guaranteed by the Mean Value Theorem for Derivatives on the interval [1,4] for the function $f(x) + g(x)$

Find $A + B + C + D$.

#6 Mu School Bowl
MAΘ National Convention 2021

$f(x)$ and $g(x)$ are continuous, twice-differentiable functions. Using only the information in the table below, find the indicated values.

$x =$	**1**	**2**	**3**	**4**
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Let D be the value of c guaranteed by the Mean Value Theorem for Derivatives on the interval [1,4] for the function $f(x) + g(x)$

Find $A + B + C + D$.

Mu School Bowl
Test #331
Question #7

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Question #7

#7 Mu School Bowl
MAΘ National Convention 2021

Let A be the number of distinct arrangements of the word *COMBINATORICS*

Let B be the number of positive integer solutions of the form (X_1, X_2, X_3) to the equation

$$X_1 + X_2 + X_3 = 2021$$

Let C be the number of terms in the expansion of $(X_1 + X_2 + X_3)^{2021}$

Jae has 16 (indistinguishable) black Math Competition shirts and 5 (indistinguishable) gold Math Competition shirts. Let D be the number of ways he can hang these shirts in his (linear) closet so that each gold shirt is separated by at least two black shirts. Gold shirts may be at the leftmost or rightmost end.

Find:

$$\frac{1}{5!}\frac{A}{D} + \frac{{}_{2023}P_5}{BC}$$

#7 Mu School Bowl
MAΘ National Convention 2021

Let A be the number of distinct arrangements of the word *COMBINATORICS*

Let B be the number of positive integer solutions of the form (X_1, X_2, X_3) to the equation

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Find:

$$\frac{1}{5!}\frac{A}{D} + \frac{{}_{2023}P_5}{BC}$$

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Question #8

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Question #8

#8 Mu School Bowl
MAΘ National Convention 2021

Let $A = y(1)$ if $y' = 2xy + 2x + y + 1$ and $y(0) = 0$

Let $B = y(1)$ if $y + xy' = 6e^{3x}$ and $y(0) = 0$

Let $C = y(1)$ if $(e^{3x} - 1)y' + 3e^{3x}y = 0$ and $\lim\limits_{x \to -\infty} y = 1$

Let $D = y(-1)$ if $(e^{3x} - 1)y'' + 6e^{3x}y' + 9e^{3x}y = 0$ and $\lim\limits_{x \to -\infty} y = 2$

Find

$$(A + 1)e + B + \frac{D}{C}$$

#8 Mu School Bowl
MAΘ National Convention 2021

Let $A = y(1)$ if $y' = 2xy + 2x + y + 1$ and $y(0) = 0$

Let $B = y(1)$ if $y + xy' = 6e^{3x}$ and $y(0) = 0$

Let $C = y(1)$ if $(e^{3x} - 1)y' + 3e^{3x}y = 0$ and $\lim\limits_{x \to -\infty} y = 1$

Let $D = y(-1)$ if $(e^{3x} - 1)y'' + 6e^{3x}y' + 9e^{3x}y = 0$ and $\lim\limits_{x \to -\infty} y = 2$

Find

$$(A + 1)e + B + \frac{D}{C}$$

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Test #331
Question #9

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Test #331
Question #9

#9 Mu School Bowl
MAΘ National Convention 2021

Let $A = \int_0^{\pi/6} x^3 \tan(x^5 + x)\,dx$ Let $B = \int_0^{\pi/6} (2x\sec(x) + \sec^2(x))dx$

Let $C = \int_0^{\pi/6} x^2 \sec(x)\tan(x)dx$ Let $D = \int_0^{\pi/6} \frac{x^3\tan(x) + x^3\tan(x^5)}{\tan(x^5)\tan(x) - 1}dx$

If $A + B + C + D = \frac{(\pi^2 + p)\sqrt{q}}{r}$, and q is square-free, find $p + q + r$

#9 Mu School Bowl
MAΘ National Convention 2021

Let $A = \int_0^{\pi/6} x^3 \tan(x^5 + x)\,dx$ Let $B = \int_0^{\pi/6} (2x\sec(x) + \sec^2(x))dx$

Let $C = \int_0^{\pi/6} x^2 \sec(x)\tan(x)dx$ Let $D = \int_0^{\pi/6} \frac{x^3\tan(x) + x^3\tan(x^5)}{\tan(x^5)\tan(x) - 1}dx$

If $A + B + C + D = \frac{(\pi^2 + p)\sqrt{q}}{r}$, and q is square-free, find $p + q + r$

Mu School Bowl
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Question #10

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Test #331
Question #10

#10 Mu School Bowl
MAΘ National Convention 2021

For each of the following series, assign a value of 21 if it converges absolutely, 20 if it converges conditionally, and -10 if it diverges. Your final answer is the sum of assigned values.

$\sum_{n=1}^{\infty} \frac{(-1)^n n^{2021}}{2021^n}$	$\sum_{n=1}^{\infty} \left(\frac{1}{4}\right)^n \frac{(2n)!}{(n!)^2}$	$\sum_{n=1}^{\infty} \frac{\sin(n)}{n}$
$\sum_{n=1}^{\infty} \frac{(-1)^n n^2}{n^2+1}$	$\sum_{n=2}^{\infty} \frac{(-1)^n}{n\ln(n)}$	$\sum_{n=1}^{\infty} \frac{(-2)^n}{n!}$
$\sum_{n=1}^{\infty} 2021^{-n-(-1)^n}$	$\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$	$\sum_{n=1}^{\infty} \frac{(-1)^n n!}{n^n}$

#10 Mu School Bowl
MAΘ National Convention 2021

For each of the following series, assign a value of 21 if it converges absolutely, 20 if it converges conditionally, and -10 if it diverges. Your final answer is the sum of assigned values.

$\sum_{n=1}^{\infty} \frac{(-1)^n n^{2021}}{2021^n}$	$\sum_{n=1}^{\infty} \left(\frac{1}{4}\right)^n \frac{(2n)!}{(n!)^2}$	$\sum_{n=1}^{\infty} \frac{\sin(n)}{n}$
$\sum_{n=1}^{\infty} \frac{(-1)^n n^2}{n^2+1}$	$\sum_{n=2}^{\infty} \frac{(-1)^n}{n\ln(n)}$	$\sum_{n=1}^{\infty} \frac{(-2)^n}{n!}$
$\sum_{n=1}^{\infty} 2021^{-n-(-1)^n}$	$\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$	$\sum_{n=1}^{\infty} \frac{(-1)^n n!}{n^n}$

Mu School Bowl
Test #331
Question #11

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Question #11

Consider the conic section $rx^2 + (1 - r)xy + y^2 + 2y - 2 = 0$ for real r.

Let A be the sum of all values of r for which this conic is a circle.

Let B be the sum of all values of r for which this conic is a parabola.

Let C be the sum of all values of r for which this conic is degenerate.

Using rotation of variables, this conic can be written in the form $A'x'^2 + C'y'^2 + D'x' + E'y' + F' = 0$ in the rotated x'-y' plane. Let θ be any angle of rotation for which this will be true for all values of $r \neq 1$, and let $D = \sin^2(\theta)\cos^2(\theta)$.

Find $A + B + CD$

#11 Mu School Bowl
MAΘ National Convention 2021

Consider the conic section $rx^2 + (1 - r)xy + y^2 + 2y - 2 = 0$ for real r.

Let A be the sum of all values of r for which this conic is a circle.

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Find $A + B + CD$

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Question #12

Mu School Bowl
Test #331
Question #12

#12 Mu School Bowl
MAΘ National Convention 2021

For $f(x) = \frac{1}{x^2}$, consider the region between $f(x)$ and the x-axis from $x = 1$ to $x = 2$.

Let A be the area of this region.

Let B be the volume of this region when it is revolved around the line $y = -1$.

Let C be the volume of this region when it is revolved around the y-axis.

Find $\frac{A \cdot C}{B}$

#12 Mu School Bowl
MAΘ National Convention 2021

For $f(x) = \frac{1}{x^2}$, consider the region between $f(x)$ and the x-axis from $x = 1$ to $x = 2$.

Let A be the area of this region.

Let B be the volume of this region when it is revolved around the line $y = -1$.

Let C be the volume of this region when it is revolved around the y-axis.

Find $\frac{A \cdot C}{B}$

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Question #13

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Test #331
Question #13

#13 Mu School Bowl
MAΘ National Convention 2021

Let A be the maximum value of $2x + 3y + 6z$ given that $x^2 + y^2 + z^2 = 16$.

Let B be the minimum possible value of $x^2 + y^2 + z^2$ given that $x + 4y + 8z = 27$.

Let C be the minimum possible value of $(x + y + z)^3$ given that $xyz = 8$ and $x, y, z > 0$.

Find the maximum possible value of xyz given that $Ax + By + Cz = 18$ and $x, y, z > 0$. Express your answer as a common fraction.

#13 Mu School Bowl
MAΘ National Convention 2021

Let A be the maximum value of $2x + 3y + 6z$ given that $x^2 + y^2 + z^2 = 16$.

Let B be the minimum possible value of $x^2 + y^2 + z^2$ given that $x + 4y + 8z = 27$.

Let C be the minimum possible value of $(x + y + z)^3$ given that $xyz = 8$ and $x, y, z > 0$.

Find the maximum possible value of xyz given that $Ax + By + Cz = 18$ and $x, y, z > 0$. Express your answer as a common fraction.

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Question #14

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Test #331
Question #14

The function $f(x) = \begin{cases} \arctan(x) & x \geq 1 \\ Ax^3 + Bx^2 + Cx + D & x < 1 \end{cases}$ is continuous, as are $f'(x)$, $f''(x)$, and $f'''(x)$.

Find $24 \cdot (A \cdot B + C + D)$.

#14 Mu School Bowl
MAΘ National Convention 2021

The function $f(x) = \begin{cases} \arctan(x) & x \geq 1 \\ Ax^3 + Bx^2 + Cx + D & x < 1 \end{cases}$ is continuous, as are $f'(x)$, $f''(x)$, and $f'''(x)$.

Find $24 \cdot (A \cdot B + C + D)$.