Theta

Applications

Test #411

Directions:

1. Fill out the top section of the Round 2 Google Form answer sheet and select **Theta-Applications** as the test. Do not abbreviate your school name. Enter an email address that will accept outside emails (some school email addresses do not).

2. Scoring for this test is 5 times the number correct plus the number omitted.

3. TURN OFF ALL CELL PHONES.

4. No calculators may be used on this test.

5. Any inappropriate behavior or any form of cheating will lead to a ban of the student and/or school from future National Conventions, disqualification of the student and/or school from this Convention, at the discretion of the Mu Alpha Theta Governing Council.

6. If a student believes a test item is defective, select "E) NOTA" and file a dispute explaining why.

7. If an answer choice is incomplete, it is considered incorrect. For example, if an equation has three solutions, an answer choice containing only two of those solutions is incorrect.

8. If a problem has wording like "which of the following could be" or "what is one solution of", an answer choice providing one of the possibilities is considered to be correct. Do not select "E) NOTA" in that instance.

9. If a problem has multiple equivalent answers, any of those answers will be counted as correct, even if one answer choice is in a simpler format than another. Do not select "E) NOTA" in that instance.

10. Unless a question asks for an approximation or a rounded answer, give the exact answer.

The legendary mathemagician Mr. Lu the Wiz has been defeated by a nameless individual from Application Nation. Since you are Mr. Lu's top disciple as well as a prodigy in mathemagics, you must avenge him. As always, NOTA stands for "None of the above." Good luck!

1. You must prepare for your journey carefully. Being a cautious individual, you decide to bring 8 visually indistinguishable mathemagical backup wands. You have 3 distinct bags you can carry your wands in. If each bag must contain at least one wand, how many ways can you put the wands in the bags?

A. 21 B. 35 C. 42 D. 45 E. NOTA

2. After you finish packing, you see something flying towards you. It's Kira, Mr. Lu's magical birb! Startled, you accidentally wave your wand four times and then drop it. You notice that the path that Kira is taking mimics a cubic function: $K(x)$. From your four wand waves, you have determined that $K(0) = 1, K(1) = 2, K(2) = 3$, and $K(3) = 5$. Find $K(4)$.

A. 6 B. 7 C. 8 D. 9 E. NOTA

3. "Salutations!" the birb says. "Hi?" you respond thinking about how you just lost your first wand. "I was informed about your journey and I would like to assist you," Kira says. "You need to travel quite a distance to reach Application Nation. With my magical birb powers, I can fly you halfway there at a rate of 80 mph, and then the rest of the way at 60 mph." If you accept Kira's offer, what will be your average speed throughout the journey? (All answers are mph)

A. 60 B. $\frac{440}{7}$ C. $\frac{480}{7}$ D. 70 E. NOTA

4. The two of you depart. Unfortunately, it seems that Kira has overestimated her magical ability and you crash-land before you reach Application Nation. If you started at (0, 0), $y = 0$ is the ground, and your flight can be modeled by $-4x^4 + 3x^3 + 8x^2 + 4x = 0$, where did you land?

A. (0, 0) B. (2, 0) C. (0, 2) D. (3, 0) E. NOTA

5. Having crash-landed, you lost your sense of direction. "Dang it! I forgot to bring a compass!" you exclaim. You take out a backup wand. "Tell me where Application Nation is!" you command. However, mathemagical wands are quite tricky. In order for the wand to be used, the user must be worthy. Each wand magically projects a question that must be solved before you can use it. This wand's question is to find the remainder when $x^4 + 3x^3 - 8x^2 + x - 1$ is divided by $x^2 - 3x + 2$.

A. 0 B. $5x - 6$ C. $10x - 11$ D. $13x - 17$ E. NOTA

6. The wand activates and shows you the way. Because each wand can only be used once, you discard it. You and the birb follow the path, but you run into a troll hiding under a log that blocks the way. "Halt!" the troll says, jumping out. "In order to pass, you will have to beat me in a game! But I only play against intelligent lifeforms, so first, answer me a few log questions. First, what number is $log_{49}(625) \cdot log_5(9) \cdot log_{16}\left(\frac{1}{7}\right) \cdot log_3(64)$ equal to?"

A. -6 B. -1 C. 1 D. 6 E. NOTA

7. "Hm…" the troll says. "It seems like you have some intellect after all. But not enough! Second, which one of the choices is a real solution for x where $(log_2(x))^4 + (log_x(2))^4 = 47$."

A. 2 B. $2^{\frac{-3-\sqrt{5}}{2}}$ C. $2^{\frac{5-\sqrt{3}}{2}}$ D. 4 E. NOTA

8. "You're good!" the troll admits. "But I have one final question! I have here a magical berry bush. It grows at a continuous exponential rate in which it will double in size after $ln(3)$ minutes. How many hours do I have to wait for it to grow to 100 times its original size?

A. $2log_3(10)$ B. $2ln(10)log_2(3)$ C. $\frac{log_3(10)}{30}$ D. $\frac{ln(10)log_2(3)}{30}$ E. NOTA

9. "Alright, I think you are worthy of playing me in a game." the troll says. "The game is simple. We will take turns taking from that pile of coins." he says pointing to a mountain of coins under the log. "Each turn, we can take anywhere from 1 to 9 coins. We cannot skip a turn. The person who takes the last coin wins. I choose to go first so you can pick how many coins we start with." "Okay." you say. Which of the following amounts of coins will guarantee that you win?

A. 2020 B. 2021 C. 2022 D. 2023 E. NOTA

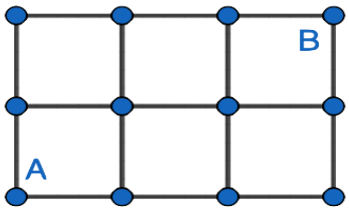
10. "NOOO!" the troll cries as you defeat him. Unfortunately, you still have to find a way to get over the log. Stumped, you decide to use another wand. This time, it's a geometry problem: you have a regular octagon *SEPTAGON* (brilliant naming, I know) with side length 1. Two squares *SEBC* and *EPDF* are drawn such that none of the three shapes' areas overlap. What is the area of triangle *BEF*?

A. $\frac{\sqrt{2}}{4}$ B. $\frac{1}{2}$ C. $\frac{\sqrt{2}}{2}$ D. 1 E. NOTA

11. With the log out of the way, you and the birb manage to reach Application Nation's outer wall. "Halt!" a guard says. *Again?* you sigh. "Who are you?" the guard asks. "Just a traveler." you respond. "Well, only qualified people are allowed to enter Application Nation, so first answer the gate question." *The what?* you think confused. Then, you notice there's some words written on the gate. It reads: There are three lines with lengths of 8, 5, and 19. How many lines with an integral length can be combined with these lines to form a quadrilateral with positive area?

A. 6 B. 7 C. 24 D. 26 E. NOTA

12. The guard lets you in. "Hm this city is pretty big. I hope I don't get lost." you say. "Don't worry," Kira says, "I found a map of the city." *Pretty strange city*. you think to yourself. You want to go from A to B. You can only travel up or to the right and you can only travel along the lines. How many distinct ways can you go from A to B?



A. 8 B. 10 C. 16 D. 20 E. NOTA

13. You reach the palace and demand an audience with the king. "Sir, we can't let anyone through." the guards tell you. You sigh and pull out a wand to put the guards to sleep. The question is: Which of the following operations are closed over all real numbers?
I: Addition II: Subtraction III: Multiplication IV: Division V: Exponentiation

A. I, II, IV B. I, II, III, IV C. I, II, III, V D. All of them E. NOTA

14. The guards fall asleep and you open the door. Unfortunately, more guards are inside. "He's a mathemagician! Subdue him!" the guards shout after seeing your wand. Panicking, you pull out another wand. This one has a probability question: You have a bag of 6 cards. You know that 4 of the cards are red on both sides, and the remaining 2 are red on one side and green on the other. You reach in and randomly pull out a card and see that one side is red. What is the probability the other side of your card is also red?

A. $\frac{1}{3}$ B. $\frac{2}{3}$ C. $\frac{3}{4}$ D. $\frac{4}{5}$ E. NOTA

15. POOF! The guards turn into iguanas. You see more guards running at you from a distance. You quickly pull out another wand with another probability problem: You are drawing cards from a shuffled, standard 52-card deck. The cards are face down and you are drawing them one at a time. What is the probability the last two cards you draw are both aces?

A. $\frac{1}{221}$ B. $\frac{2}{221}$ C. $\frac{223}{15744}$ D. $\frac{3}{51}$ E. NOTA

16. You successfully take down those guards as well, but see even more coming. “How many guards does this place have?!” you exclaim. Not wanting to waste any more wands, you and the birb duck into a room. “Gah!” Kira shouts. You see that the room you entered has a square hole in the floor. A rope is attached to a pole at one corner of the square with Kira attached to the other. You can tell that if Kira falls into the square hole, bad things will happen. Some quick measurements tell you that the square has an area of 75 square feet and the rope is 10 feet long. Given that the room is quite large and that Kira cannot step into the hole, what is the area of the place Kira is confined to? All answers are in square feet. (ignore the fact that Kira can fly and any unmentioned measurements, such as Kira’s height)

A. $\frac{150\pi-100\sqrt{3}+175}{2}$ B. $\frac{275\pi-75\sqrt{3}}{3}$ C. $\frac{150\pi-100\sqrt{3}}{2}$ D. $\frac{275\pi-75\sqrt{3}}{2}$ E. NOTA

17. You tug on the rope but it doesn’t budge. You take out another wand. If $2x + 3y = 12, y + 3z = 18,$ and $z + 2x = 6,$ what is the value of $x + y + z$?

A. 0 B. 5 C. 7 D. 9 E. NOTA

18. The birb is freed and you two listen as the guards run past the room. Cautiously, you stick your head out the door. The guards have passed the door already so you decide to make a run for it. You and Kira start running/flying down the 100 foot long corridor. Immediately, the guards notice and start chasing you, but they are 20 feet behind where you started. When Kira reaches the end of the corridor, you are 5 feet behind her and the guards are 10 feet behind you. Assuming everyone moves at constant speeds, how far behind will the guards be after you reach the end of the corridor? All answers in feet.

A. $\frac{160}{19}$ B. $\frac{170}{19}$ C. $\frac{180}{19}$ D. $\frac{185}{19}$ E. NOTA

19. Barely escaping the guards, you shut the door on them. In the room, you see an infinite cauldron with some Super-Duper-All-Curing-Antidote-2000, except the measurements are a bit off. *Hm… this feels eerily familiar…*. The infinite cauldron contains 20 liters of 10% magic, 30% health, and 60% water. You know the correct concentrations are 25% magic, 25% health, and 50% water. Looking around, you see three infinite bottles, one with 80% magic 20% water, another with 90% health and 10% water, and a third with 50% magic and 50% health. What is the least number of liters of liquids you must add to the cauldron to get the correct concentrations? All answers are in liters.

A. $\frac{11}{2}$ B. $\frac{17}{3}$ C. 6 D. 8 E. NOTA

20. You take some of the antidote with you just in case. You realize that you are stuck in the room and the only way out is back through the door that you trapped the guards behind. Sighing, you pull out a wand. It asks: What is the sum of the reciprocals of the roots of $2x^5 - 3x^4 - x^3 - 27x^2 - x + 30 = 0$ for all real x?

A. $\frac{1}{3}$ B. $-\frac{1}{3}$ C. $\frac{3}{2}$ D. 30 E. NOTA

21. As you are powering through the seemingly endless amount of guards, your mind starts to wander. If you can defeat the first guard in $\frac{1}{2}$ a minute, the second in $\frac{1}{3}$ a minute, the third in $\frac{1}{6}$ a minute, the fourth in $\frac{2}{27}$ of a minute, the fifth in $\frac{5}{162}$ of a minute, and that pattern continues infinitely, how long will it take you to defeat all of the guards? All of the answers are in minutes. (Kira whispers to you: don't take all of the fractions at face value and look for a pattern)

A. $\frac{3}{4}$ B. 1 C. $\frac{9}{8}$ D. No solution E. NOTA

22. Of course, there aren't actually infinite guards so you finish them off quicker than expected. You walk into the throne room to see the king, Hector the Vector. You tell the king your story but he is not impressed. You pull out your last backup wand to convince Hector to listen to your demands. The question goes: You are traveling with a birb on the Cartesian plane. You are currently standing at the point (-6, 2) and your destination is at (2, 10). However, there is a road represented by $y = x - 2$ which you must go to first. What is the least distance you can travel to reach your destination?

A. $\sqrt{82}$ B. $2\sqrt{82}$ C. $8\sqrt{2}$ D. $16\sqrt{2}$ E. NOTA

23. Finally, the king allows you to battle the one who took down Mr. Lu: the most high-IQ individual of Application Nation. "You want to go against me in a battle of the wits?" the high-IQ individual asks. "Yeah" you reply. "Okay then. You're probably quite frustrated with the amount of guards we had so here's a fitting question: What is the volume of a frustum with radii of 6 and 10 with a slant height of 5?

A. 98π B. 126π C. 196π D. 252π E. NOTA

24. "Hm. Not bad." the high-IQ individual says. "How about this one then: You have a triangle *ABC* with side lengths of $2\sqrt{13}, \sqrt{3},$ and 7. What is the sum of the triangle's area and its circumradius?

A. $\frac{4\sqrt{13}+7\sqrt{3}}{2}$ B. $\sqrt{13} + 7\sqrt{3}$ C. $\frac{2\sqrt{26}+7\sqrt{3}}{2}$ D. $\frac{2\sqrt{13}+7\sqrt{3}}{2}$ E. NOTA

25. “Okay, you’ve got some skill. How about this one then? I have a rectangular prism with a length of 3 feet, a width of 4 feet, and a height of 12 feet. What is the length of the longest rod that I can fit into my prism?” (Ignore the thickness of the rod) All answers are in feet.

A. 7 B. 13 C. 49 D. 169 E. NOTA

26. “Brilliant!” the high-IQ individual exclaims. “Okay then, here’s the last problem I have prepared for you. Imagine that we are at a party with 30 people. I tell you to split the people into groups, and you split them into groups of 4, 5, 6, 7, 8. I then decide I needed 3 people to assist me with a task. So I pick one of the five groups you have made, then 3 people from that group. How many ways could I have done this?”

A. 63 B. 125 C. 126 D. 127 E. NOTA

27. “Haha. You’re good,” the infinite-IQ individual says. “Now, I assume you also have questions for me?” “Yeah,” you reply. “One of my friends owns some chikadees and horsees. Chikadees have 1 head and 2 legs each, while horsees have 1 head and 4 legs each. If my friend counts a total of 8 heads and 28 legs, how many horsees does he have?” Solve your own problem to verify if the infinite-IQ individual is correct!

A. 2 B. 4 C. 6 D. 8 E. NOTA

28. “That one was too easy.” the infinite-IQ individual says. “Okay, how about this one then?” you reply. “I am mailing letters to 5 people. I have 5 distinct letters and 5 distinct envelopes. However, I accidentally put all of the letters in the envelopes randomly. How many ways are there for me to have put all of the letters in the wrong envelope?”

A. 41 B. 42 C. 43 D. 44 E. NOTA

29. “Not hard enough.” the infinite-IQ individual says as he answers correctly again. “How about this question? It’s similar to the one that brought Mr. Lu the Wiz down. Which step of this proof is not a logical continuation of the previous step?” “Ha!” shouts Kira. “I was there too! You can’t trick me twice! It’s:” What is the answer?

$$1 = \sqrt{1} \quad (1)$$
$$1 = \sqrt{(-1)(-1)} \quad (2)$$
$$1 = \sqrt{-1} * \sqrt{-1} \quad (3)$$
$$1 = i * i \quad (4)$$
$$1 = -1 \quad (5)$$

A. 2 B. 3 C. 4 D. 5 E. NOTA

30. “Well, well, well. You’re really good.” says the infinite-IQ individual. “If you can’t beat ‘em, join ‘em. Why don’t we start a new kingdom? We can call it High-IQ-opolis. What do you say?” “Okay?” you say with much uncertainty. But you are sure that you cannot defeat the infinite-IQ individual. “I have one last test for you mathemagician.” the infinite-IQ individual says. “I have 7 apples. You take 2 of them. How many apples do you have?”

A. 4 B. 5 C. 6 D. 7 E. NOTA